What is irrational about irrationality?

J. Acacio de Barros

Liberal Studies Program
San Francisco State University

Ubiquitous Quanta, Università del Salento
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What is rationality?

- "To accept something as rational is to accept it as making sense, as appropriate, or required, or in accordance with some acknowledged goal, such as aiming at truth or aiming at the good.[1]"

[1] S. Blackburn, *The Oxford Dictionary of Philosophy*. Oxford, UK: Oxford University Press, 2nd ed., 2005

Outline

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Adaptation matters

Figure : *Paranthropos Boisei*

But mind is an adaptation

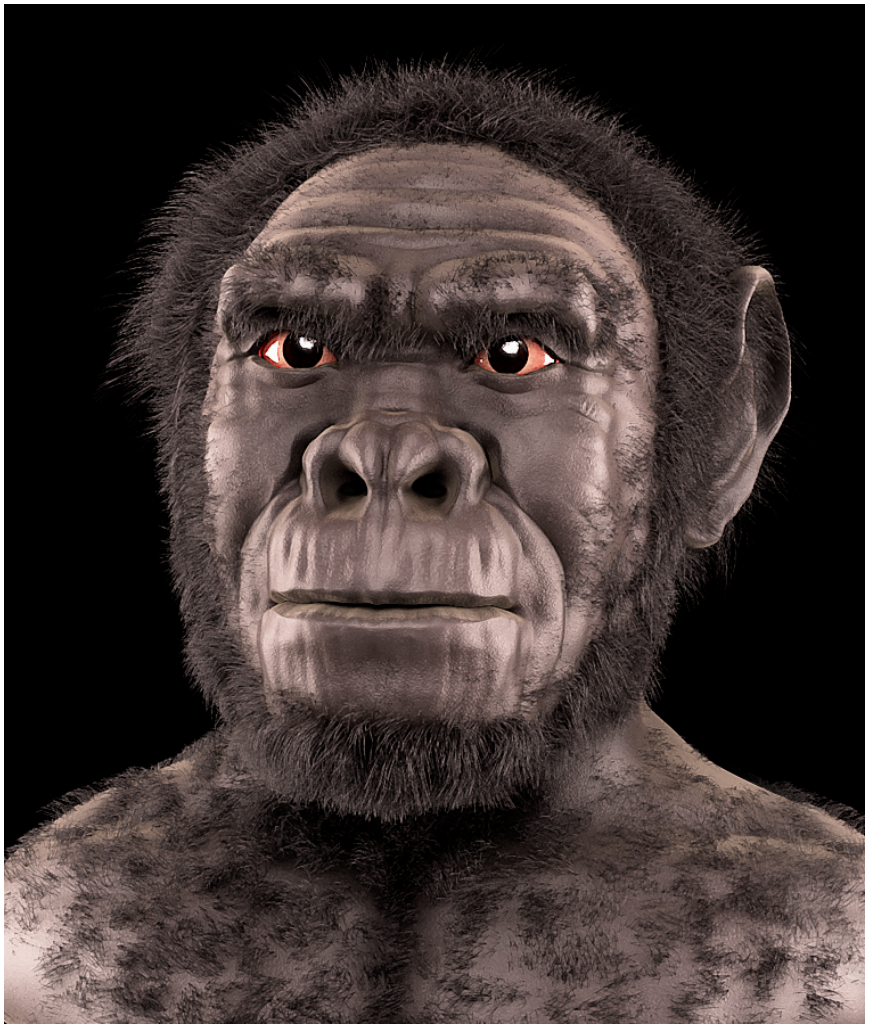

Figure : *Homo Habilis*

A little bit later

Figure : *Homo sapiens*

Early Astronomy

- *Homo sapiens:*
  - 195-160 ka (bottleneck of ~600 before 145 ka); Fossil evidence suggests migration to marine shellfish collection.
  - "Being an efficient human shellfish collector requires the novel connection of lunar patterns to tidal variation to shellfish return rates and safety of collection, substantial planning abilities, and communication of complex parameters between group members. All of this is a signal that the enhanced working memory and executive functions [...] of the modern human intellect are in place."[2]

[2] C. W. Marean, "Coastal south africa and the co-evolution of the modern human lineage and the coastal adaptation," in *Trekking the Shore: Changing Coastlines and the Antiquity of Coastal Settlement* (N. Bicho, J. A. Haws, and L. G. Davis, eds.), pp. 421–440, New York: Springer, 2011

Origins of modern Logic

- Stories need to make sense.
- We can use logic to pinpoint flaws or contradictions in a story.
- Aristotle's logic was in published in his *Organon*, but is related to his *Rhetoric*.
- "A deduction is speech (*logos*) in which, certain things having been supposed, something different from those supposed results of necessity because of their being so." (Aristotle, *Prior Analytics* I.2, 24b18-20)

Importance of consistency

- A contradiction is a statement of the form $A \wedge (\neg A)$
  - E.g. "God is compassionate and God is not compassionate."
- Contradictions are something that bother us.
- Classic logic show us why:
  - If $A \wedge \neg A$ is true, then A is true, and so is $\neg A$.
  - If A is true, for **any** B, $A \vee B$ is also true.
  - If $\neg A$ and $A \vee B$ are true, then B must be true!

Logic of uncertainty?

- Logic tells us what we can rationally deduct from what we know to be true.
- But what about what we believe to be true, but do not know for sure?
- How do we deal with things that are uncertain?
- We use probabilities.

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Origins of probabilities

- Probabilities had their origin from Pascal, a Jansenist studying payoffs on games of chance (a contradiction?).
- Probabilities are **normative** (not descriptive). I.e., logicists proposed probabilities as measures of "rational" belief.
  - The probability that an event is three to one means "that in the universal opinion of those who examine this subject, the state of mind to which a person *ought* to be able to bring himself is to look three times as confidently upon the arrival as upon the non-arrival" (De Morgan)
  - "Probability I conceive as to be not so much expectation, as a rational ground for expectation." (Boole)

Kolmogorov's axioms of probability

Let Ω be a set, and $\mathcal{F}$ an σ-algebra of subset on Ω, and $P : \mathcal{F} \to \mathbb{R}$. Then $(\Omega, \mathcal{F}, P)$ is a probability space if and only if

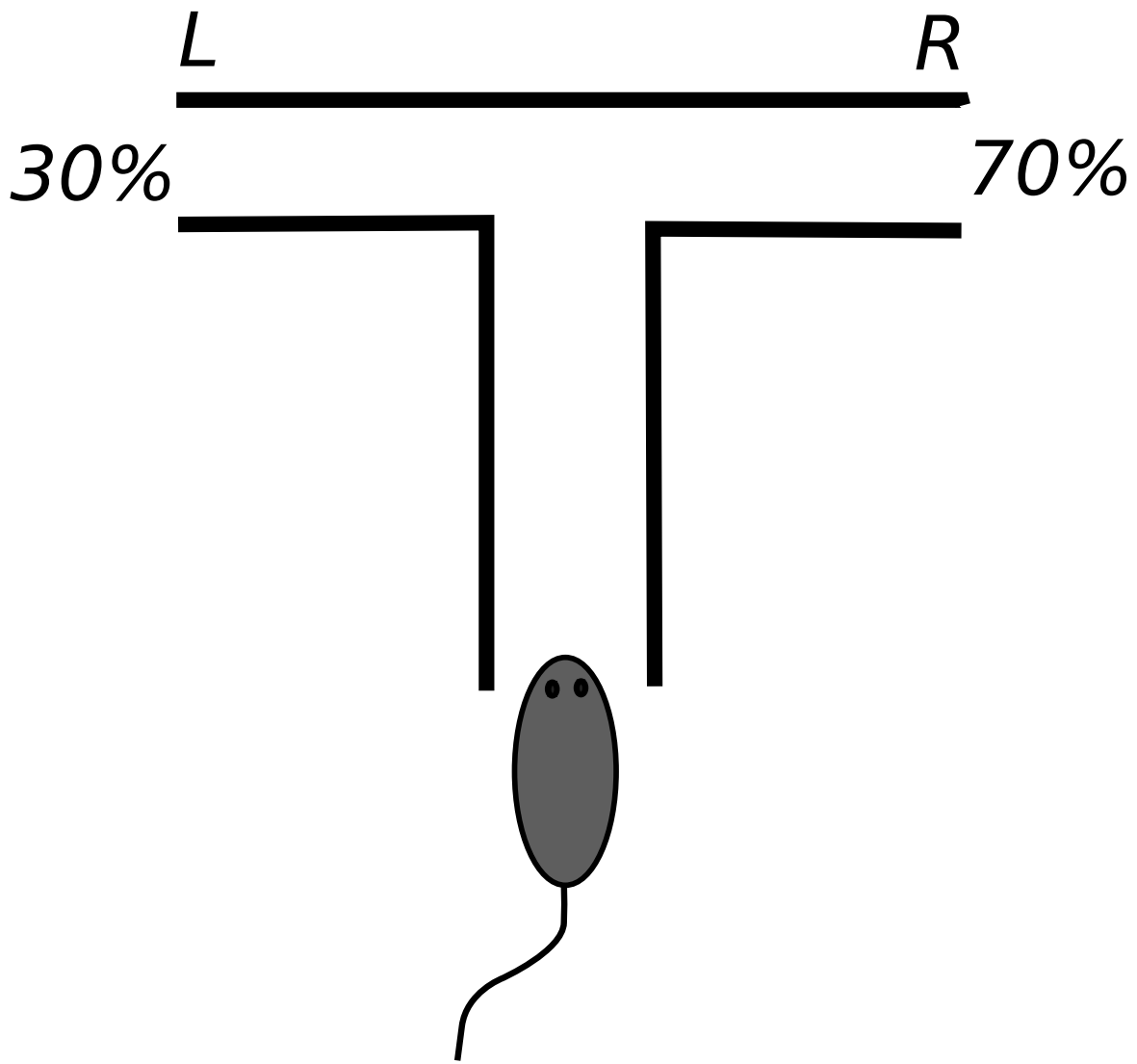
- (i) $P(A) \geq 0$, $A \in \mathcal{F}$,
- (ii) $P(\Omega) = 1$,
- (iii) $P(A \cup B) = P(A) + P(B)$ for $A \cap B = \emptyset$.

Rational thought led us far

- Galileo:
  - The book of nature is written in mathematical language.
  - Conceptual representation has to be consistent not only with itself but with *empirical evidence*.
  - We are telling a consistent story about the real (observable) world.
- Newton (stood on Galileo's shoulder):
  - Pushed mathematical ideas to extreme.
- Darwin
- Maxwell
- Einstein
- Bohr?

Outline

Our first hint: are rats rational?



What is wrong with rats?

- Rats are not "rational:" they choose the middle ground.
  - In that sense, rats are closer to politicians than to scientists.
- But is the maximization of utility the best option for rats?
  - As a species, perhaps not: maximization assumes stationarity
- Same for humans: experiments also show probability matching (though in different contexts).
  - But is the maximization of utility the best option for humans? Tragedy of the commons!

Other types or irrationality

Disjunction fallacy:

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Which is more probable?

1. Linda is a bank teller.
2. Linda is a bank teller and is active in the feminist movement.

More of the same?

Violation of Savage's STP:

Context 1:

- Pass class, buy ticket? (Most prefer to buy)
- Fail class, buy ticket? (Most prefer to buy)

Context 2:

- Don't know, buy ticket? (Most prefer **not** to buy!)

Context is everything

- It is tempting to label the above instances as irrational.
- However, each question (Linda or Buying tickets) are highly contextual.
  - In Linda's case, we tell a story beforehand.
  - In buying tickets, we tell students they either passed a class or not (context 1) or we simply tell them they do not know (context 2).
- As in the case of the rats, our brains are taking other things into consideration.

Contextuality and Heisenberg

- Ironically, contextuality may be illustrated by an example from Heisenberg's personal history: his Copenhagen visit in 1941.
  - Margrethe (Bohr's wife): "Why did he [Heisenberg] come to Copenhagen?"
  - Heisenberg (Frayn's play Copenhagen): "No one understands my trip to Copenhagen. Time and time again I've explained it. To Bohr himself, and Margrethe. To interrogators and intelligence officers, to journalists and historians. The more I've explained, the deeper the uncertainty has become. Well, I shall be happy to make one more attempt."
- Facts are often contradictory, usually in very subtle ways.
- A story's meaning is often constructed; it does not come uniquely from "facts."

Outline

How to describe irrationality?

- The essence of mathematics is to express logical thinking.
- How do we describe irrational information?
- Here we mean two things (more details below):
  - Reasoning that does not follow the standard rules of inference (logic/probability).
  - Reasoning that is made from conflicting information (from multiple contexts).

Reasoning with nonstandard rules of inference

- Let A be "Linda is a bank teller," and B "Linda is a bank teller and is active in the feminist movement."
- Since $B \subseteq A$, $B \subseteq A$. Then define $B' = A \setminus B$. Then B and B' are disjoint, and $B \cup B' = A$.
- But for disjoint sets

$$P\left(B \cup B'\right) = P(B) + P(B') = P\left(A\right).$$

From axiom A1 (positivity), it follows that if $B \subseteq A$ then $P(B) \leq P(A)$.

- Kolmogorov implies monotonicity, but some people reason nomonotonicaly.

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Reasoning from conflicting information

- Let $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{Z}$ be ± 1 random variables with zero expectation representing future trends on stocks of companies X, Y, and Z going up or down.
- Three experts, Alice, Bob, and Carlos, have beliefs about the relative behavior of pairs of stocks.
- There is no probability space that describes this system if experts give the following expectations: $E_A(\mathbf{XY}) = -1$, $E_B(\mathbf{XZ}) = -1/2$, $E_C(\mathbf{YZ}) = 0$.[3]

[3] J. A. d. de Barros, "Decision making for inconsistent expert judgments using negative probabilities," in *Quantum Interaction 2013 Conference Proceedings*, (Leicester, UK), 2013. To appear

Contextuality is key

- In both cases, one could expand the probability space to include contextual cues.
- For example, instead of insisting that our probability space for $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{Z}$ is only made up of three random variables, we could instead talk about $\mathbf{X}_A$ for Alice, and $\mathbf{X}_B$ for Bob. Unnatural!
- So, how to deal with contextuality in a more "natural" way?

Quantum models.

- Since classical probability theory seems to be violated by human behavior, cognitive scientists looked for alternatives to it.
- One such alternative is the use of the mathematical apparatus of quantum mechanics to model human behavior.
- This comes from the well-known fact that quantum mechanics violate Kolmogorov's axioms.
- Example: the double slit experiment
  - How can we have

$$P(\text{slit } 1) > P(\text{slit } 1 \cup \text{slit } 2)?$$

- Non-monotonicity is dealt in QM with superpositions of quantum states (quantum interference).

What do we need from QM?

- Nonlocality.
- Contextuality.
- Indeterminism.

- For social phenomena, we do not need nonlocality.[4]
- Can we explore the constraints of QM formalism? Are there alternatives?

[4] J. A. de Barros and P. Suppes, "Quantum mechanics, interference, and the brain," *Journal of Mathematical Psychology*, vol. 53, no. 5, pp. 306–313, 2009.

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One alternative: nonstandard probabilities

- One possible alternative is to use alternative measures of probability that do not conform with Kolmogorov's axioms.
- Examples are:
  - Upper probabilities (Bruno De Finetti)
  - Negative probabilities (with minimization of negative mass)
- Both cases above may exhibit non-monotonicity, and can be applied to quantum systems where no probability distribution exists[5].

[5] J. A. de Barros and P. Suppes, "Probabilistic inequalities and upper probabilities in quantum mechanical entanglement," *Manuscrito*, vol. 33, pp. 55–71, 2010; P. Suppes and M. Zanotti, "Existence of hidden variables having only upper probabilities," *Foundations of Physics*, vol. 21, no. 12, pp. 1479–1499, 1991

Can we justify negative probabilities?

- Cortical neurons have interference-like collective effects.[6]
- Such effects could be related to non-monotonic quantum-like effects, such as the violation of Savage's STP.[7]
  - They can be described using negative probabilities.
- Since we evolved to deal quickly with inconsistent information, it is a reasonable assumption that we reason (intuitively) with negative probabilities, as it is computationally cheap.

[6] P. Suppes, J. A. de Barros, and G. Oas, "Phase-oscillator computations as neural models of stimulus–response conditioning and response selection," *Journal of Mathematical Psychology*, vol. 56, pp. 95–117, Apr. 2012

[7] J. A. de Barros, "Quantum-like model of behavioral response computation using neural oscillators," *Biosystems*, vol. 110, pp. 171–182, Dec. 2012

Normative or descriptive?

- Probabilities provide a way to make rational decisions. However, they have problems when inconsistent information is provided (no joint probability function exists).
- Negative probabilities or a Hilbert space formalism do not suffer the same problem; they may be used to deal with such inconsistencies.
- Negative probabilities can provide the best set of decisions that approach rationality.[8]

[8] J. A. d. de Barros, "Decision making for inconsistent expert judgments using negative probabilities," in *Quantum Interaction 2013 Conference Proceedings*, (Leicester, UK), 2013. To appear

Summary

- Rational descriptions are essential in decision-making since our early (evolutionary) origins.
- But we are often subject to inconsistent information, due to multiple contexts or contradictory opinions.
- We evolved to deal with inconsistent information (e.g., more food on the right but also understanding of non-stationarity of environment) in a quick fashion, and this might have left us with residual irrational ways of thinking.
- Irrational reasoning or reasoning with inconsistent information may be accomplished with a Hilbert space formalism or with extended probabilities (among other approaches).
- Alternative formalisms beyond standard probability theory, such as quantum-like ones or extended probabilities, may provide a glimpse into how we think, as well as furnish norms on how to be as rational as possible when a purely rational approach is not possible.

Thank you!