Some Examples of Contextuality in Physics

and its implications to quantum cognition

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Weird Quantum Mechanics

Bohr

Anyone not shocked by quantum mechanics has not yet understood it.

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Anyone not shocked by quantum mechanics has not yet understood it.

Feynman

Nobody understands quantum mechanics.

Weird Quantum Mechanics

Bohr

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Feynman

Nobody understands quantum mechanics.

Those quotes are representative of quantum mechanics:

- They seem contradictory.
- They are contextual.

Contextuality at the core

- Bohr complementarity principle: QM is contextual.
- But what types of contextuality?
  - Double-slit
  - Kochen-Specker
  - Bell
  - GHZ

Contextuality at the core

- Bohr complementarity principle: QM is contextual.
- But what types of contextuality?
  - Double-slit
  - Kochen-Specker
  - Bell
  - GHZ
- In this talk we will discuss what is different about each of them, and what we can learn that informs us about using QM mathematical tools outside of QM (e.g., in quantum cognition).

Outline

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What is contextuality?

- $\mathbf{P}$, $\mathbf{Q}$, and $\mathbf{R}$ are the outcomes of measuring three properties.
- We only observe them in three different contexts:$(\mathbf{P},\mathbf{Q})$, $(\mathbf{P},\mathbf{R})$, or $(\mathbf{Q},\mathbf{R})$.
- The existence of a joint probability distribution for $(\mathbf{P},\mathbf{Q},\mathbf{R})$ consistent with the pairwise expectations imply that $\mathbf{P}$ in context $(\mathbf{P},\mathbf{Q})$ is the same as $\mathbf{P}$ in context $(\mathbf{P},\mathbf{R})$, and so for $\mathbf{R}$ and $\mathbf{Q}$.
- Contextuality corresponds to a lack of joint probability distribution.

Double-slit: what is it?

double_slit_electrons_setup.jpg[1]

double_slit_

[1]Copyright, Hitachi Co. http://www.hitachi.com/rd/portal/research/em/doubleslit.html.

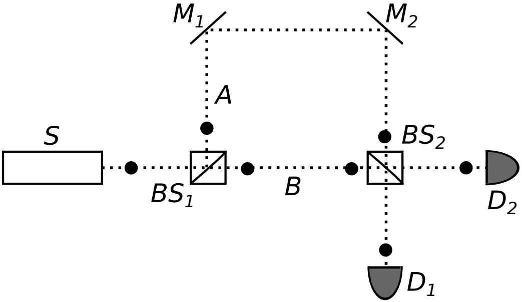
Double slit: simplified version

MachZehnderSimple.svg

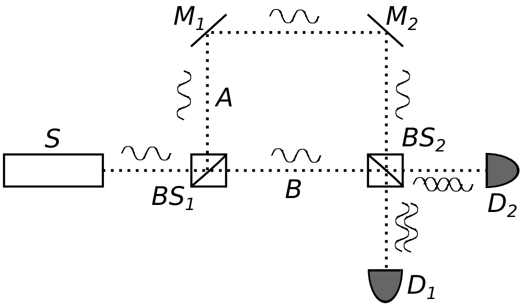
double_slit_electrons_setup.jpg[2]

[2] Copyright, Hitachi Co.
http://www.hitachi.com/rd/portal/research/em/doubleslit.html.

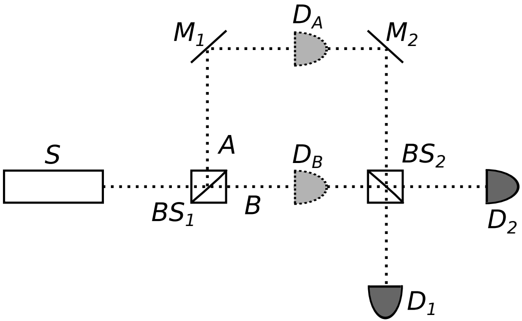
Double slit: particles



Double slit: waves



Double slit: the mystery?



Double slit: according to Feynman

We choose to examine a phenomenon which is impossible, absolutely impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality, it contains the only mystery. We cannot explain the mystery in the sense of "explaining" how it works. -Feynman

Double slit: no joint probabilities

- The double slit presents extreme contextuality: $\mathbf{D}_1$ and $\mathbf{D}_2$ depend on whether we measure which-path information ($\mathbf{D}_A$ or $\mathbf{D}_B$).
- Collapse changes wave function at $\mathbf{D}_1$ and $\mathbf{D}_2$.
- This contextuality is manifest in the non-monotonicity, and in the lack of a joint.
- Feynman[3] (and Scully[4]) remark that the nonmonotonicity due to interference can be modeled by negative probabilities (they can't[5]).

[3] Feynman, R. (1987) Negative probability In B.J. Hiley and F.D. Peat, (ed.), Quantum implications: essays in honour of David Bohm, pp. 235–248 Routledge London and New York

[4] Scully, M. O., Walther, H., and Schleich, W. March 1994 *Physical Review A* **49(3)**, 1562–1566

[5] Oas, G., deBarros, J. A., and Carvalhaes, C. (2014) *Physica Scripta* In Press. arXiv:1404.3831 [quant-ph], deBarros, J. A. and Oas, G. April 2014 *arXiv:1404.3921 [physics, physics:quant-ph]*

Kochen-Specker: Cabello's version

For any state[6]:

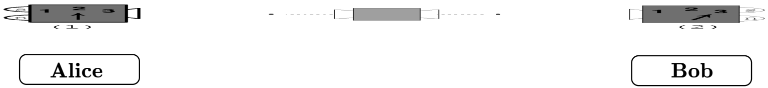
$$
\begin{aligned}
\mathbf{V}_{0,0,0,1} + \mathbf{V}_{0,0,1,0} + \mathbf{V}_{1,1,0,0} + \mathbf{V}_{1,-1,0,0} &= 1, \\
\mathbf{V}_{0,0,0,1} + \mathbf{V}_{0,1,0,0} + \mathbf{V}_{1,0,1,0} + \mathbf{V}_{1,0,-1,0} &= 1, \\
\mathbf{V}_{1,-1,1,-1} + \mathbf{V}_{1,-1,-1,1} + \mathbf{V}_{1,1,0,0} + \mathbf{V}_{0,0,1,1} &= 1, \\
\mathbf{V}_{1,-1,1,-1} + \mathbf{V}_{1,1,1,1} + \mathbf{V}_{1,0,-1,0} + \mathbf{V}_{0,1,0,-1} &= 1, \\
\mathbf{V}_{0,0,1,0} + \mathbf{V}_{0,1,0,0} + \mathbf{V}_{1,0,0,1} + \mathbf{V}_{1,0,0,-1} &= 1, \\
\mathbf{V}_{1,-1,-1,1} + \mathbf{V}_{1,1,1,1} + \mathbf{V}_{1,0,0,-1} + \mathbf{V}_{0,1,-1,0} &= 1, \\
\mathbf{V}_{1,1,-1,1} + \mathbf{V}_{1,1,1,-1} + \mathbf{V}_{1,-1,0,0} + \mathbf{V}_{0,0,1,1} &= 1, \\
\mathbf{V}_{1,1,-1,1} + \mathbf{V}_{-1,1,1,1} + \mathbf{V}_{1,0,1,0} + \mathbf{V}_{0,1,0,-1} &= 1, \\
\mathbf{V}_{1,1,1,-1} + \mathbf{V}_{-1,1,1,1} + \mathbf{V}_{1,0,0,1} + \mathbf{V}_{0,1,-1,0} &= 1.
\end{aligned}
$$

[6] Cabello, A., Estebaranz, J., and Alcaine, G. March 1996 *Physics Letters A* **212(4)**, 183–187 arXiv:quant-ph/9706009

Kochen-Specker: no joint probabilities

- Measurements are state independent.
- We cannot assign values 0 or 1 to observables that are independent of context.
    - Assuming that $\mathbf{V}_{0,0,0,1}$ is the same in context $(\mathbf{V}_{0,0,0,1}, \mathbf{V}_{0,0,1,0}, \mathbf{V}_{1,1,0,0}, \mathbf{V}_{1,-1,0,0})$ as in context $(\mathbf{V}_{0,0,0,1}, \mathbf{V}_{0,1,0,0}, \mathbf{V}_{1,0,1,0}, \mathbf{V}_{1,0,-1,0})$ for all projectors lead to inconsistencies.

EPR: the setup



EPR: no joint probabilities

- Alice measures either $\mathbf{A}_1$ or $\mathbf{A}_2$.
  Bob measures either $\mathbf{B}_1$ or $\mathbf{B}_2$.
- $[\hat{A}_1, \hat{B}_1] = [\hat{A}_1, \hat{B}_2] = [\hat{A}_2, \hat{B}_1] = [\hat{A}_2, \hat{B}_2] = 0$, $[\hat{A}_1, \hat{A}_2] \neq 0 \neq [\hat{B}_1, \hat{B}_2]$.
- For certain (entangled) quantum states (and certain observables), there exists no joint probabilities for $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{B}_1$, and $\mathbf{B}_2$.
- In other words, we cannot assume that $\mathbf{A}_i$ is the same in contexts $(\mathbf{A}_i, \mathbf{B}_1)$ and $(\mathbf{A}_i, \mathbf{B}_2)$.
- Additionally, because the system is bipartite, Alice and Bob's measures can be spacelike separated.

GHZ: The setup

GHZ.eps

GHZ: No joint probabilities

- Alice measures either $\hat{\sigma}_x$ or $\hat{\sigma}_y$. Same for Bob or Carlos.
- For certain entangled states: $\left\langle \hat{\sigma}_x^{(A)} \hat{\sigma}_x^{(B)} \hat{\sigma}_y^{(C)} \right\rangle = \left\langle \hat{\sigma}_x^{(A)} \hat{\sigma}_y^{(B)} \hat{\sigma}_x^{(C)} \right\rangle = \left\langle \hat{\sigma}_y^{(A)} \hat{\sigma}_x^{(B)} \hat{\sigma}_x^{(C)} \right\rangle = 1$, but $\left\langle \hat{\sigma}_y^{(A)} \hat{\sigma}_y^{(B)} \hat{\sigma}_y^{(C)} \right\rangle = -1$.
- No joint: $\left(\mathbf{X}^{(A)}\mathbf{X}^{(B)}\mathbf{Y}^{(C)}\right)\left(\mathbf{X}^{(A)}\mathbf{Y}^{(B)}\mathbf{X}^{(C)}\right)\left(\mathbf{Y}^{(A)}\mathbf{X}^{(B)}\mathbf{X}^{(C)}\right) = \left(\mathbf{Y}^{(A)}\mathbf{Y}^{(B)}\mathbf{Y}^{(C)}\right)$.
- Contradiction.

GHZ: where does it stand?

- GHZ is similar Kochen-Specker, in the sense that a contradiction is derived.
- Similar to EPR, as nonlocality is involved (tri-partite system).
- It is also state dependent.

Different "types" of contextuality

- Double-slit experiment.
  - On same run, either measure two incompatible experiments or not.
  - Measuring which-path alters $|\psi\rangle$ and affects $\mathbf{D}_1$ and $\mathbf{D}_2$ (signaling).
  - Contextuality comes from signaling (violation of marginal selectivity).

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  - Contextuality comes from signaling (violation of marginal selectivity).
- Kochen-Specker.
  - On same run, measures several *compatible* observables.
  - Contextuality comes from the impossibility to assign values to observables consistent with all different experimental conditions.

Different "types" of contextuality

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  - Contextuality comes from signaling (violation of marginal selectivity).
- Kochen-Specker.
  - On same run, measures several *compatible* observables.
  - Contextuality comes from the impossibility to assign values to observables consistent with all different experimental conditions.
- EPR and GHZ.
  - Similar to Kochen-Specker; on same run measures compatible observables.
  - Difference is that contextuality holds on multi-partite systems, and measurements can be spacelike separated (nonlocality).

Outline

The conjunction fallacy

- Violation of classical probability theory[7]

$$P(\text{Linda is a bank teller}) < P(\text{Linda is a bank teller AND a feminist})$$

- Can be modeled by a quantum formalism[8]:
  - "Bank teller" and "feminist" are non-orthogonal vectors in a 2-dim space.
  - State of mind is represented by a $|\psi\rangle$, which collapses first into the observable associated to "feminist" and then into "bank teller."
  - Similar to double slit.

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7 Tversky, A. and Kahneman, D. (1983) *Psychological Review* **90(4)**, 293–315

8 Busemeyer, J. R., Pothos, E. M., Franco, R., and Trueblood, J. S. (2011) *Psychological Review* **118(2)**, 193–218

Violation of Savage's STP

- From the monotonicity of probability, if $P(A|B) > x$ and $P(A|\neg B) > x$, then $P(A) > x$.
- Human decision makers violate the STP [9].
- Has also been modeled by quantum formalism[10].
- As the conjunction fallacy, collapse is used, similar to the double slit.

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[9] Tversky, A. and Shafir, E. September 1992 *Psychological Science* **3(5)**, 305–309, Shafir, E. and Tversky, A. October 1992 *Cognitive Psychology* **24(4)**, 449–474

[10] Pothos, E. M. and Busemeyer, J. R. (2009) *Proceedings of the Royal Society B: Biological Sciences* **276(1665)**, 2171–2178

Order effects

- Gallup pools have shown order effects:
  - "Is Clinton honest?" and then "Is Gore honest?" gives different results than if asked Gore and then Clinton.
- Order effects were modeled by quantum formalism[11].
  - The model assumes a vector representing "Clinton is honest" as not orthogonal to "Gore is honest".
  - The state of mind $|\psi\rangle$ collapses first onto either Clinton honest/not-honest and then onto Gore in one order, but reversed in the other order.
- As the above examples, collapse is used, similar to the double slit.

[11] Wang, Z. and Busemeyer, J. R. October 2013 *Topics in Cognitive Science* 5(4), 689–710

Other examples

- Other examples exist of QM models in cognition[12].
- It is unclear whether such models present different contextuality than the two slit.
    - Some may violate the no-signaling condition.

[12] See Khrennikov, A. (2010) Ubiquitous Quantum Structure, Springer Verlag, Heidelberg, Busemeyer, J. and Bruza, P. (2012) Quantum models of cognition and decision, Cambridge Univ. Press, Cambridge, or Haven, E. and Khrennikov, A. (2013) Quantum Social Science, Cambridge Univ. Press, Cambridge.

Outline

Negative probabilities: natural for physical systems?

- Negative probabilities have a long history in physics[13].
- According to Feynman, the double slit is an interference of negative and positive probabilities. Negative-valued probabilities would explain the non-monotonic character of the system.
- However, no NP exists for it [14]; stronger contextuality.
- On the other hand, for other examples we have negative probabilities.

[13] Wigner, E. June 1932 *Physical Review* **40(5)**, 749–759, Dirac, P. (1942) *Proceedings of the Royal Society of London B* **A180**, 1–40

[14] deBarros, J. A. and Oas, G. April 2014 *arXiv:1404.3921 [physics, physics:quant-ph]*

Contextuality by default: more general

- Contextuality by default: for the three variable case, instead $\mathbf{P}$, $\mathbf{Q}$, and $\mathbf{R}$ for the three contexts,$(\mathbf{P},\mathbf{Q})$, $(\mathbf{P},\mathbf{R})$, or $(\mathbf{Q},\mathbf{R})$, we have $\mathbf{P_Q}$, $\mathbf{P_R}$, $\mathbf{Q_P}$, $\mathbf{Q_R}$, $\mathbf{R_P}$, and $\mathbf{R_Q}$.
- There is always a joint for the CbD variables[15].
- Describes all examples (allows for signaling) [16].

[15] Dzhafarov, E. and Kujala, J. May 2013 *PLoS ONE* **8(5)**, e61712

[16] Dzhafarov, E. and Kujala, J. July 2014 *arXiv:1407.2886 [quant-ph]*

NP: measure of contextuality

- Both NP and CbD provide a way to measure contextuality[17].
- In NP, the L1 norm of the probabilities

$$M = \sum |P(\omega)|$$

measures how NP departs from a proper joint.

  - For $M > 1$ there is contextuality.
  - For $M = 1$ there is no contextuality (proper joint).
  - The minimum value of M provides a measure of contextuality.

[17] deBarros, J. A., Dzhafarov, E., Kujala, J., and Oas, G. June 2014 *arXiv:1406.3088 [quant-ph]* arXiv: 1406.3088

CbD: measure of contextuality

- In CdB, each random variable is indexed by context.
- No contextuality means that a given variable $\mathbf{A}_{1,1}$ in context 1 can be the same as $\mathbf{A}_{1,2}$ in context 2, i.e. it is possible to find a joint such that $P(\mathbf{A}_{1,1} \neq \mathbf{A}_{1,2}) = 0$.
- The minimum value of the quantity

$$\Delta = \sum_{j \neq j'} P\left(\mathbf{A}_{i,j} \neq \mathbf{A}_{i,j'}\right)$$

provides a measure of contextuality (0 for a noncontextual).

Hilbert spaces

- Contextual systems can be described by the tools of QM.
- However, not everything goes for QM systems.
  - E.g. QM systems do not allow certain correlations (PR boxes).
  - Quantum boundary is still not explained by basic principles.

Contextuality in perception

- Consider the example to be shown.
  - Ask yourself if they are correlated (i.e., both are lit or dark at the same time) or if they are anti-correlated (when one is lit, the other is dark).

Mathematical description

- The example seem can be described by three pairwise anti-correlated random variables **X**, **Y**, and **Z**.
- They are contextual, as there is no joint[18].

[18] Suppes, P. and Zanotti, M. (1981) *Synthese* **48(2)**, 191–199

[19] deBarros, J. A. December 2012 In A. Khrennikov, A. L Migdall, S. Polyakov, and H. Atmanspacher, (ed.), AIP Conference Proceedings, volume **1508**, Vaxjo, Sweden: American Institute of Physics. pp. 98–107, deBarros, J. A. September 2013 *arXiv:1309.3775 [physics, physics:quant-ph, q-bio]*, deBarros, J. A. (2014) Decision making for inconsistent expert judgments using negative probabilities Lecture Notes in Computer Science pp. 257–269 Springer Berlin/Heidelberg

Mathematical description

- The example seem can be described by three pairwise anti-correlated random variables **X**, **Y**, and **Z**.
- They are contextual, as there is no joint[18].
- However, **X**, **Y**, and **Z** is not a quantum mechanical system![19]

[18] Suppes, P. and Zanotti, M. (1981) *Synthese* **48(2)**, 191–199

[19] deBarros, J. A. December 2012 In A. Khrennikov, A. L Migdall, S. Polyakov, and H. Atmanspacher, (ed.), AIP Conference Proceedings, volume **1508**, Vaxjo, Sweden: American Institute of Physics. pp. 98–107, deBarros, J. A. September 2013 *arXiv:1309.3775 [physics, physics:quant-ph, q-bio]*, deBarros, J. A. (2014) Decision making for inconsistent expert judgments using negative probabilities Lecture Notes in Computer Science pp. 257–269 Springer Berlin/Heidelberg

Summary

- Contextuality comes in physics in different flavors.
  - Double slit
  - EPR/GHZ
  - Kochen-Specker
- The most relevant to psychology is the double-slit experiment.
- However, QM also brings lots of restrictions.
  Could other tools (such as NP or CbD) be better for cognitive models?

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The end

Thank you!