Contextuality, and Language

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Outline

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1. Motivation
2. Contextuality in Physics
3. Contextuality in Language
   - Truth and Belief

Contextuality in Physics

- Contextuality is a central concept in foundations quantum physics
- It is a mathematically well-defined concept in physics

Contextuality in Linguistics

- In linguistics contextuality is a key concept in semantics and in pragmatics
- The concept of context in linguistics precedes that of physics
  - Abner Shimony (1968)
  - P. A. Heelan. "Complementarity, context dependence, and quantum logic", *Foundations of Physics*, **1**(2), 95-110, (1970).
- However, contrary to physics, there is no well-defined unifying concept of contextuality.

Contextuality in Linguistics

- "Suppose someone suspects that an expression *e* [...] is context-sensitive. How could he go about establishing this? One way that philosophers of language do so is to think about (or imagine) various utterances of sentences containing *e*. If they have intuitions that a semantically relevant feature of those utterances varies from context to context, then that, it is assumed, is evidence [that] *e* is context-sensitive." (Cappelen & Lepore 2008)
  - If we restrict ourselves to formal semantics, then how do we think about "intuitions" about variation with context?
  - Given an utterance, what are the conditions under which we must call such utterance contextual?
  - How could we go and establish that *e* is contextual?

Connecting Physics and Linguistics

- The question we want to answer is this:
  - Is contextualty in physics connected to linguistic contextuality?
  - Can our tools from physics help with linguistics?

Outline

Kochen-Specker

- The most famous example of contextuality in physics was given by Kochen and Specker.
  - For a Hilbert space of dimension 3:
    - a set of projection operators, P_i, corresponding to true or false propositions about the physical system
    - sets of contexts where *some* of those P_i's are compatible
    - no context-independent truth-values can be assigned to the outcomes of such measurements in all contexts
- In other words, we cannot assign a truth value to a P_i that is the same in one context as in another; the property P_i depends on the context.

A non-physical example of KS-type contextuality

- To understand what we mean by contextuality, let us examine a simple example (not possible in physics).
  - Let P_1, P_2, and P_3 be two-valued variables, either 1 (true) or 0 (false).
  - Let the following be three contexts: $C_1 = (P_1, P_2)$, $C_2 = (P_1, P_3)$, and $C_3 = (P_2, P_3)$.
  - Let the following be true for each context:

$$P_1 + P_2 = 1,$$
$$P_1 + P_3 = 1,$$
$$P_2 + P_3 = 1.$$

However, we can see that there is an inconsistency: even on the left, odd on the right.

Inconsistency comes from assuming non-contextuality

- Contradiction comes from assuming that truth-values are the same in each context.
  - Say $P_1 = 1$ and $P_2 = 0$ in the context C_1, and $P_1 = 1$ and $P_3 = 0$ in context C_2.
  - But we know that in C_3 either $P_2 = 0$ and $P_3 = 1$ or $P_2 = 1$ and $P_3 = 0$, i.e. one of them is not the same as in the other contexts.
- That is why we say that the random variables in this example are contextual.

Another example with perfect correlation

- GHZ state
$$\frac{1}{\sqrt{2}}\left(|+++\rangle - |---\rangle\right).$$
- Observables $\hat{A} = \sigma_x^{(1)}\sigma_y^{(2)}\sigma_y^{(3)}$, $\hat{B} = \sigma_y^{(1)}\sigma_x^{(2)}\sigma_y^{(3)}$, $\hat{C} = \sigma_y^{(1)}\sigma_y^{(2)}\sigma_x^{(3)}$, and $\hat{D} = \sigma_x^{(1)}\sigma_x^{(2)}\sigma_x^{(3)}$.
- From QM, $E\left(\hat{A}\right) = E\left(\hat{B}\right) = E\left(\hat{C}\right) = -E\left(\hat{D}\right) = 1$, and from the algebra $\hat{D} = \hat{A}\hat{B}\hat{C}$.
- But $E\left(D\right) = -1 = E\left(ABC\right) = E\left(A\right)E\left(B\right)E\left(C\right) = 1$, a contradiction.
- However, in this case it is non-local.

Extension to non-perfect correlation

- Cases above were for perfect correlations: not a realistic assumption.
- For imperfect correlations, we need to use probabilities.
- But for non-contextual variables, logical entailements lead to probabilities satisfying certain inequalities.

An example with imperfect correlation

- Suppes-Zanotti inequality

  $$-1 \leq \langle \mathbf{AB} \rangle + \langle \mathbf{AC} \rangle + \langle \mathbf{BC} \rangle \leq 1 + 2\min\{\langle \mathbf{AB} \rangle, \langle \mathbf{AC} \rangle, \langle \mathbf{BC} \rangle\},$$

  where **A**, **B**, and **C** are ± 1-valued random variables.
- To see that this must be the case, we can examine all logical possibilities for each product:

$$(\mathbf{AB} = 1 \& \mathbf{AC} = 1) \rightarrow \mathbf{BC} = 1$$

$$(\mathbf{AB} = 1 \& \mathbf{AC} = -1) \rightarrow \mathbf{BC} = -1$$

$$(\mathbf{AB} = -1 \& \mathbf{AC} = 1) \rightarrow \mathbf{BC} = -1$$

$$(\mathbf{AB} = -1 \& \mathbf{AC} = -1) \rightarrow \mathbf{BC} = 1.$$

- Since each line above add to numbers that are either -1 or 3, their convex combination must be greater than -1.

Imperfect correlation for GHZ

- For the GHZ example above, we have A, B, C, and $D = ABC$ as our simplified set of random variables.
- Let us examine the following logical possibilities:

$$(A = 1 \& B = 1 \& C = 1) \rightarrow D = 1$$
$$(A = 1 \& B = 1 \& C = -1) \rightarrow D = -1$$
$$\vdots$$
$$(A = -1 \& B = -1 \& C = -1) \rightarrow D = -1.$$

- A convex sum of all those possibilities imply that

$$-2 \leq E(A) + E(B) + E(C) - E(D) \leq 2$$

(and permutations of the $-$ sign).
- Those are necessary and sufficient conditions for existence of a joint.

Summarizing contextuality in physics

- Physical systems are contextual: in some cases the value of a property cannot be the same in all experimental contexts
  - When properties are "true" or "false," contextuality manifests as a logical contradiction.
  - When the properties are stochastic, contextuality manifests as the absence of a joint probability. The logical contradiction is a special case when probabilities are one.

Outline

Basic semantics

- Assumption: linguistic meanings need to be logically consistent.
  - We will presume a theory of meaning, i.e. a way to attribute truth values to an utterance.
  - We impose on this theory standard rules of reasoning
- Participants in a linguistic exchange are trying to express ideas that make sense (to them)
  - if they are inconsistent, how can we even try to attribute truth-values and meaning?

Indexicals

- Indexicals are possibly the most discussed form of contextuality in linguistics.
  - To assign a truth value an uttered proposition we need to know:
    - who uttered it,
    - when they uttered it, and
    - whether it is actually true under such conditions.

Indexicals

Example

- Bob is a graduate student at UC Berkeley, and he likes to surf and go to the beach. Because it is California (suck it Indiana!), Bob went to the beach on November 9th, 2018.
- Alice is also a graduate student, but she goes to Stanford. Alice is Bob's friend. When he invited her to go to the beach on November 9th, 2018, she declined, stating that she had plans to go to the movies with Carlos instead. Bob was devastated.

$P =$ "I went to the beach yesterday."

Context 1. On the evening of November 10th, 2018, P is uttered by Alice.

Context 2. On the morning of November 10th, 2018, P is uttered by Bob.

Indexicals are contextual

- The contextuality of P comes from being uttered in two different contexts
  - They are two different propositions!

P1. "I went to the beach yesterday," uttered by Alice at 7:32pm of November 10th, 2018 sipping a coffee in Palo Alto.

P2. "I went to the beach yesterday," uttered by Bob at 2:11am of November 10th, 2018 drinking a beer at Antonio's Nut House.

A different example

- Consider $P =$ “Aristotle knew very little philosophy.”

  Alice: P is true.

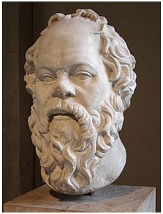
  Bob: P is false.

A different example

$P =$ "Socrates knew very little philosophy."

Alice: P is true.

Bob: P is false.

 $\neq$ 

And another example

- In addition to indexicals, there are other types of contextuality in language.
- "Do your homework" has different meanings
  - told to my kids
  - told to a colleague.
- When told to my kids, it means that the proposition "I want you to sit your butt down and finish the homework that your teacher assigned" is true, whereas the proposition "I believe you need to educate yourself more about this subject" has not necessarily an assigned truth value.

Ancillaries

- As a final example, let us consider the two following contexts:

  Context 1: Carlos says $P_1 =$"I now pronounce you men and wife" to Alice and Bob in a bar.

  Context 2: Carlos says $P_2 =$"I now pronounce you men and wife" in a formal setting of a wedding ceremony, where Carlos is the officiating person.

- In both contexts, the question for the truth value of P_1 and P_2 is somewhat irrelevant (i.e. whether Carlos is pronouncing or not). However, the following ancillary proposition is true in one context and not in the other.
  - $P_A =$ "Alice and Bob are henceforth legally married."

Pragmatics (Abamsky)

- Meaning changes with intonation as context:
  - This is a great party
  - Bob is really smart
  - You really know your stuff

Common character

- All the examples above have a common character:
- if we were to attach truth values to the same sentences (and their ancillary propositions) in different contexts, we would reach contradictions
    - Contextuality is coded by changes of truth-value from one context to another

Belief in speech

- Speech may be based on beliefs, not truth
- Propositions need to be spoken in terms of probabilities, not truth-values
- E.g. when Alice and Bob talk to each other, their goal is to communicate, share information.
  - As such, neither are completely sure (100% probability) of what the other mean: Socrates is the philosopher or footballer
  - Furthermore, beliefs can change in the course of the conversation (Bayesian update?)
- For belief (and truth) construction, there is an underlying assumption of rationality

Rational belief

- If Alice holds a belief that the utterance U is true, then "not U" must be false.
- If Alice believes that utterances U_1, U_2, ..., U_n are true, all its logical entailments should also be true
  - e.g. that "U_1 and U_2" and "$(\neg U_1 \vee U_2)$" also true, and so on.
- If Alice is uncertain about $U\prime$, she may consider the two "universes" (or consequences) where in one $U\prime$ is true and another where $U\prime$ is false, together with the logical consequences of such truth values.
- Underlying assumption: belief or truth value constructions must be done in a consistent way

Probabilities as rational belief

- How do we construct a theory of rational belief?
- From Cox's theorem:
  - If we assume a Boolean algebra of propositions and a measure p of belief consistent with it, then p must satisfy the following:
    - $0 \leq p(U) \leq 1$
    - $p(U) = 1$ if U is known for certain to be true. $p(U') = 0$ if U' is know for certain to be false.
    - For U_1 and U_2, $p(U_1 \vee U_2) = p(U_1) + p(U_2) - p(U_1 \wedge U_2)$.
  - Those give us standard probability theory.

Belief construction in language

- Rational belief construction (also in language) assumes rational updates: probability theory.
- If Alice is trying to understand Bob's utterances in a rational way, by assuming that Bob is rational, then Alice should construct meaning using an approach consistent with probability theory.
- As in physics, an utterance is contextual if there is no joint probability distribution.
    - The solution for Alice is to re-label (reinterpret) her variables accordingly

Summary

- Contextuality in language is essential for semantics and pragmatics, but it is not well defined.
- Physics and psychology present some mathematically well-defined and conceptually clear definitions of contextuality.
- We argued that the concept of contextuality in physics and psychology is applicable to linguistics.
  - However, only CbD is general enough to deal with language
- As a note, such uses might have some interesting consequences not only for linguistics, but also to philosophy of mind.

It is about time!

Thank you!!