Beyond a quantum formalism: consequences of a neural-oscillator model to quantum cognition

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Beyond quantum cognition

- Many authors discuss quantum-like effects in the social sciences[1], particularly in psychology.
- Such effects are often modeled using the quantum mechanical formalism of Hilbert spaces.
- Here we will explore some limitations that such formalism impose, and propose alternatives and possible verification.

[1] E.g. Khrennikov, A. (2010) Ubiquitous Quantum Structure, Springer Verlag, Heidelberg, Busemeyer, J. R. and Bruza, P. D. (2012) Quantum models of cognition and decision, Cambridge University Press, Cambridge, Great Britain, Haven, E. and Khrennikov, A. (2013) Quantum Social Science, Cambridge University Press, Cambridge, UK.

Outline

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What is quantum?

Three main points stick out as non-classical:[2]

- Nondeterministic.
- Contextual.
- Nonlocal, but non-signaling.

[2] E.g. deBarros, J. A. and Suppes, P. (2009) *Journal of Mathematical Psychology* **53(5)**, 306–313.

Determinism and predictability

- Classical systems can be completely unpredictable (e.g., three-body system, Sinai billiards).
- We cannot distinguish a deterministic from a stochastic dynamics.
- Should we care anyway?

Contextuallity

- Example: $[\hat{P}, \hat{Q}] \neq 0$.

[3] E.g. Moore, D. W. April 2002 *The Public Opinion Quarterly* **66(1)**, 80–91.

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- Not surprising in social sciences.

Example [3]:

- Do you generally think Bill Clinton is honest and trustworthy?
- Do you generally think Al Gore is honest and trustworthy?

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Example [3]:

- Do you generally think Bill Clinton is honest and trustworthy?
- Do you generally think Al Gore is honest and trustworthy?

- If Clinton precedes Gore: Clinton 50%, Gore 60%
- If Gore precedes Clinton: Clinton 57%, Gore 68%

[3] E.g. Moore, D. W. April 2002 *The Public Opinion Quarterly* **66(1)**, 80–91.

Nonlocality

- Nonlocality is about contextuality at-a-distance.
- But to show nonlocality, we need to show contextual relations that cannot be explained by signaling.
- Signals can, in principle, travel within the brain within 10^{-9} s.

What about the brain?

- Stochastic.
- Contextual.
- Nonlocal?

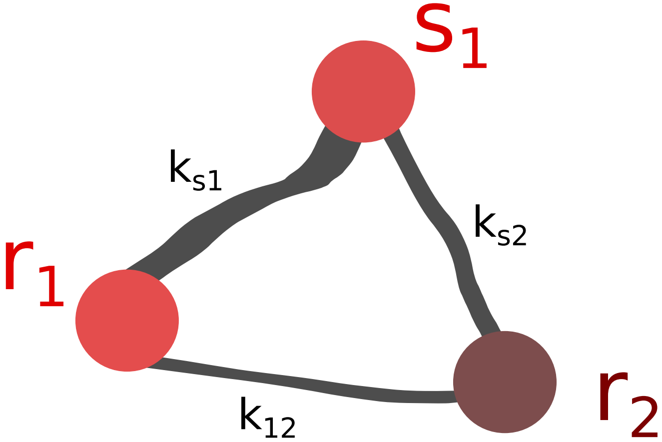
What is quantum in SS? An example

- Should I buy a plot of land given the uncertainties due to the presidential elections?
- If Republican, I decide it is better to buy.
- If Democrat, I also decide it is better to buy.
- Therefore, I should prefer buying over not buying, even if I don't know who will win (Savage's Sure-thing Principle)
- Tversky and Shafir showed that people violate the Sure-thing Principle[4].

[4] E.g. Tversky, A. and Shafir, E. September 1992 *Psychological Science* **3(5)**, 305–309, , Shafir, E. and Tversky, A. October 1992 *Cognitive Psychology* **24(4)**, 449–474.

Outline

Stimulus and response neurons



[5]

[5] Suppes, P., deBarros, J. A., and Oas, G. April 2012 *Journal of Mathematical Psychology* **56(2)**, 95–117

How to represent responses with few oscillators?

- We describe a neural oscillator's dynamics by a phase φ.[6]

$$s(t) = A_s \cos \varphi_s(t) = A_s \cos(\omega t),$$

$$r_1(t) = A_1 \cos \varphi_{r_1}(t) = A \cos(\omega t + \delta\varphi),$$

$$r_2(t) = A_2 \cos \varphi_{r_2}(t) = A \cos(\omega t + \delta\varphi - \pi).$$

$$I_1 \equiv \left\langle (s(t) + r_1(t))^2 \right\rangle_t = A^2 (1 + \cos(\delta\varphi)).$$

$$I_2 \equiv \left\langle (s(t) + r_2(t))^2 \right\rangle_t = A^2 (1 - \cos(\delta\varphi)).$$

- A response is the balance between the strengths I_1 and I_2,

$$b = \frac{I_1 - I_2}{I_1 + I_2} = \cos(\delta\varphi)$$

[6] Suppes, P., deBarros, J. A., and Oas, G. April 2012 *Journal of Mathematical Psychology* **56**(2), 95–117

Encoding responses

- To encode responses, we need to modify

$$\dot{\varphi}_i = \omega_i - \sum_{j \neq i} A_{ij} \sin(\varphi_i - \varphi_j)$$

to include angles, i.e.,

$$\dot{\phi}_i = \omega_i + \sum A_{ij} \sin(\phi_j - \phi_i + \delta\varphi_{ij}).$$

$$\dot{\phi}_i = \omega_i + \sum [A_{ij} \sin(\phi_j - \phi_i) + B_{ij} \cos(\phi_j - \phi_i)].$$

Reinforcing oscillators

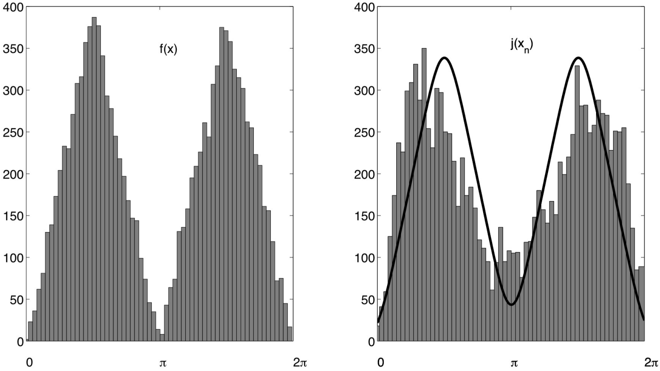
- During reinforcement:

$$\dot{\phi}_i = \omega_i + \sum \left[A_{ij} \sin(\phi_j - \phi_i) + B_{ij} \cos(\phi_j - \phi_i) \right] + K_0 \sin(\varphi_E - \varphi_i + \delta_{Ei}).$$

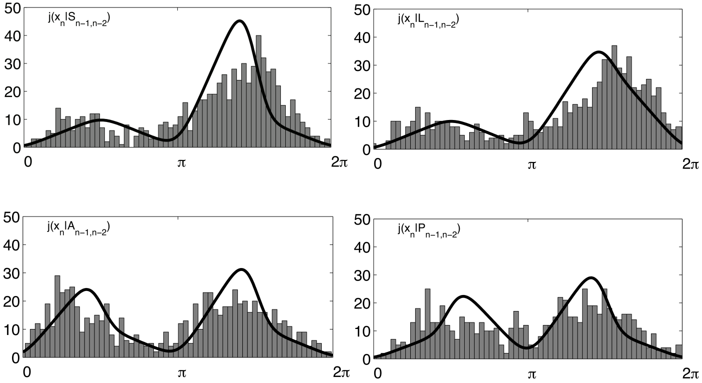
$$\frac{dk_{ij}^E}{dt} = \epsilon(K_0) \left[\alpha \cos(\varphi_i - \varphi_j) - k_{ij} \right],$$

$$\frac{dk_{ij}^I}{dt} = \epsilon(K_0) \left[\alpha \sin(\varphi_i - \varphi_j) - k_{ij}^I \right].$$

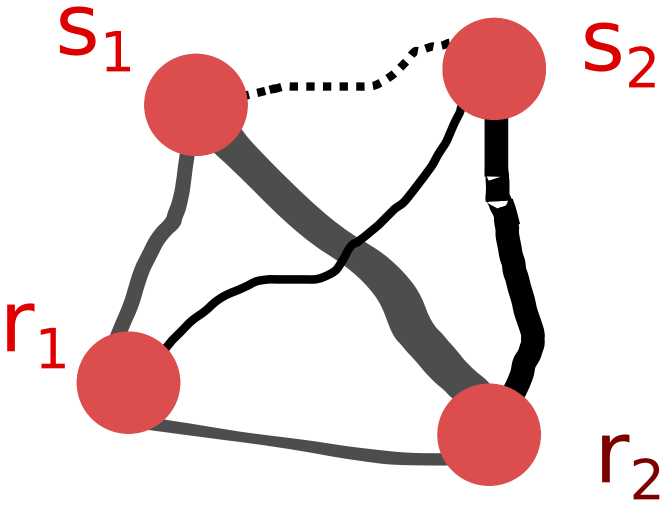
Response selection



Conditional probabilities



Oscillator interference - model



Some simulation results

- For two stimulus oscillators, s_1 and s_2, and two response oscillators, r_1 and r_2, we show "interference" of oscillators.[7]
- Well-known case of quantum-like decision making: Violation of Savage's Sure-Thing-Principle or of Kolmogorov's probability.

[7] deBarros, J. A. December 2012 *Biosystems* **110(3)**, 171–182

What the \#\$ *! do we know!?

- Propagation of oscillations on the cortex behave like a wave.
- Neural oscillator interference may be sensitive to context.
- Could quantum-like effects be simply contextual?

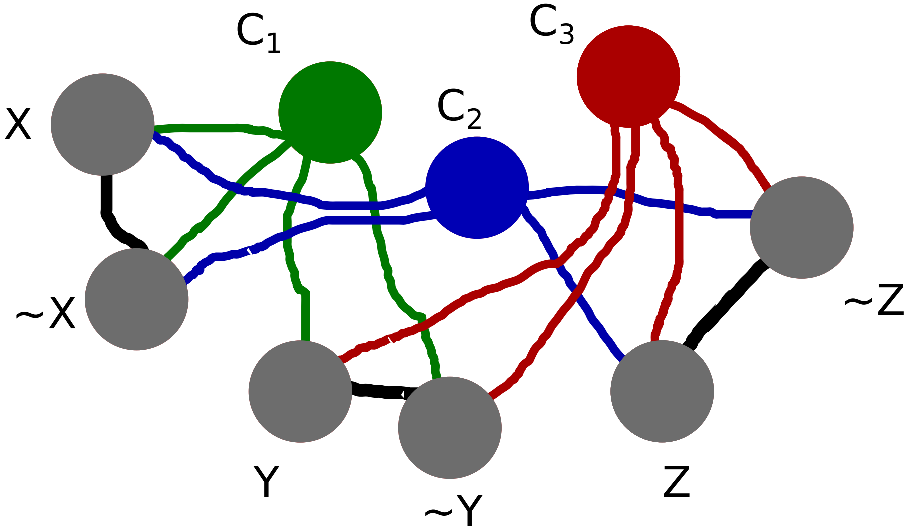
Outline

Non-trivial contextuality

- Let **X**, **Y**, and **Z** be ± 1 random variables with zero expectation.
- Let
$$E(\mathbf{XY}) = E(\mathbf{YZ}) = E(\mathbf{XZ}) = \epsilon.$$
- **X**, **Y**, and **Z** have a joint probability distribution if and only if $\epsilon > -1/3$.[8]
- This is the simplest non-trivial example of a set of random variables without a joint probability.

[8] Suppes, P. and Zanotti, M. (1981) *Synthese* 48(2), 191–199

A not-so-simple oscillator model



But it is not quantum!

- In their quantum version, we would have observables in a Hilbert space corresponding to each random variable **X**, **Y**, and **Z**. Call them $\hat{X}$, $\hat{Y}$, and $\hat{Z}$.
- To say that the correlations $E(\mathbf{XY})$, $E(\mathbf{YZ})$, and $E(\mathbf{XZ})$ are observable, means that $[\hat{X},\hat{Y}]=[\hat{X},\hat{Z}]=[\hat{Z},\hat{Y}]=0$.
- Theorem: Given three pairwise-commuting observables, $\hat{X}$, $\hat{Y}$, and $\hat{Z}$, i.e. $[\hat{X},\hat{Y}]=[\hat{X},\hat{Z}]=[\hat{Z},\hat{Y}]=0$, such that their eigenvalues are ± 1, there exists a joint probability distribution that accounts for all correlations of such observables.
- Therefore, it is possible to measure simultaneously $\hat{X}$, $\hat{Y}$, and $\hat{Z}$, which means that there exists a joint probability distribution for **X**, **Y**, and **Z**.

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A possible experiment

- The case of complete symmetry is not the most interesting.
- Instead, at each trial participants are presented with pairs of stimuli drawn from a set of three, sampled such that

$$\begin{aligned} -1 \leq\ & E(\mathbf{XY}) + E(\mathbf{YZ}) + E(\mathbf{XZ}) \leq \\ & 1 + 2\min\{E(\mathbf{XY}), E(\mathbf{YZ}), E(\mathbf{XZ})\}, \end{aligned}$$

  e.g. $E(\mathbf{XY}) = -2/3$, $E(\mathbf{XY}) = -1/3$, and $E(\mathbf{XY}) = -1/2$.
- After learning correlations, participants are asked to predict the triple moment (requires joint).
- Measurements of the triple moment could distinguish different models, such as the oscillator one presented above, an extended probability model, or a bayesian model.

Outline

Summary

- A small number of phase oscillators may be used to model a continuum of responses (with results similar to SR theory).
- The model is simple enough such that we can easily understand physically how responses are selected via inhibitory and excitatory neuronal couplings.
- Interference may help us understand how complex neural networks have "quantum-like" dynamics.
- A simple experiment can be performed to test such interference in the simplest case of three inconsistently correlated random variables.