Neural-oscillator models of quantum-decision making

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Neurons all the way down?

- What scale should we use?
  - Down to the synapse level?
  - Neurons?
  - Collective behavior of neurons?
- For language processing, robustness and measurable macroscopic effects suggest a *large* number of neurons.
- Even for a large collection of neurons, we still have several options with respect to modeling.
  - Do we need detailed interactions between neurons? Are the shapes of the action potential relevant? Timing?
- Our goal is to reduce the number of features, yet retain a physical meaning.

Outline

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SR Theory

- $(\Omega, \mathcal{F}, P)$ is a probability space
- **Z**, **S**, **R**, and **E** are random variables:
  - $\mathbf{Z} : \Omega \rightarrow E^{|S|}$
  - $\mathbf{S} : \Omega \rightarrow S$,
  - $\mathbf{R} : \Omega \rightarrow R$,
  - $\mathbf{E} : \Omega \rightarrow E$
- SR theory has the following structure:

$$\mathbf{Z}_n \rightarrow \mathbf{S}_n \rightarrow \mathbf{R}_n \rightarrow \mathbf{E}_n \rightarrow \mathbf{Z}_{n+1}. \quad (1)$$

Conditioning Axioms

C1. For each stimulus s there is on every trial a unique conditioning distribution, called the smearing distribution, which is a probability distribution on the set of possible responses.

C2. If a stimulus is sampled on a trial, the mode of its smearing distribution becomes, with probability θ, the point (if any) which is reinforced on that trial; with probability $1-\theta$ the mode remains unchanged.

C3. If no reinforcement occurs on a trial, there is no change in the smearing distribution of the sampled stimulus.

C4. Stimuli which are not sampled on a given trial do not change their smearing distributions on that trial.

C5. The probability θ that the mode of the smearing distribution of a sampled stimulus will become the point of the reinforced response is independent of the trial number and the preceding pattern of occurrence of events.

General comments on SR theory - I

(i) $\mathbf{Z}_n$ sums up previous conditioning and does not represent a computation on trial n;

(ii) $\mathbf{S}_n$ (experimentally unobserved sampling of stimuli) uses an assumption about the number of stimuli being sampled in an experiment (usually uniform);

(iii) $\mathbf{X}_n$ represents the first brain computation in the temporal sequence on a trial for the stochastic model; this computation selects the actual response on the trial from the conditioning distribution $k_s(x|z_{s,n})$, where s is the sampled stimulus on this trial $(\mathbf{S}_n = s)$;

General comments on SR theory - II

(iv) $\mathbf{Y}_n$ is the reinforcement random variable whose distribution is part of the experimental design;

(v) $\mathbf{Z}_{n+1}$ summarizes the assumed brain computations that often change at the end of a trial the state of conditioning of the stimulus s sampled on trial n; in our stochastic model, this change in conditioning is represented by a change in the mode $z_{s,n}$ of the distribution $K_s\left(x|z_{s,\,n}\right)$. .

Sampling Axioms

S1. Exactly one stimulus is sampled on each trial.

S2. Given the set of stimuli available for sampling on a given trial, the probability of sampling a given element is independent of the trial number and the preceding pattern of occurrence of events.

Response Axiom

R1. The probability of the response on a trial is solely determined by the smearing distribution of the sampled stimulus.

Outline

Stimulus and response neurons

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The intuition

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Kuramoto Equations

- If no interaction,

$$O_i(t) = A_i \cos \varphi_i(t) = A_s \cos(\omega t),$$

$$\varphi_i = \omega_i t + \delta_i,$$

and

$$\dot{\varphi}_i = \omega_i.$$

How to represent responses with few oscillators?

- Each neural oscillator's dynamics can be described by the phase, φ.

$$s(t) = A_s \cos\varphi_s(t) = A_s \cos(\omega t),$$

$$r_1(t) = A_1 \cos\varphi_{r_1}(t) = A\cos(\omega t + \delta\varphi),$$

$$r_2(t) = A_2 \cos\varphi_{r_2}(t) = A\cos(\omega t + \delta\varphi - \pi).$$

$$I_1 \equiv \left\langle (s(t) + r_1(t))^2 \right\rangle_t = A^2(1 + \cos(\delta\varphi)).$$

$$I_2 \equiv \left\langle (s(t) + r_2(t))^2 \right\rangle_t = A^2(1 - \cos(\delta\varphi)).$$

- A response is the balance between the strengths I_1 and I_2,

$$b = \frac{I_1 - I_2}{I_1 + I_2} = \cos(\delta\varphi)$$

Kuramoto Equations

- If no interaction, $\varphi_i = \omega_i t + \delta_i$, and

$$\dot{\varphi}_i = \omega_i.$$

- If we have a weak interaction, then

$$\dot{\varphi}_i = \omega_i - \sum_{j \neq i} A_{ij} \sin\left(\varphi_i - \varphi_j\right).$$

Encoding responses

- To encode responses, we need to modify

$$\dot{\varphi}_i = \omega_i - \sum_{j \neq i} A_{ij} \sin\left(\varphi_i - \varphi_j\right)$$

to include angles, i.e.,

$$\dot{\phi}_i = \omega_i + \sum A_{ij} \sin\left(\phi_j - \phi_i + \delta\varphi_{ij}\right).$$

$$\dot{\phi}_i = \omega_i + \sum \left[A_{ij} \sin\left(\phi_j - \phi_i\right) + B_{ij} \cos\left(\phi_j - \phi_i\right)\right].$$

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Reinforcing oscillators

- During reinforcement:

$$\dot{\phi}_i = \omega_i + \sum \left[A_{ij} \sin\left(\phi_j - \phi_i\right) + B_{ij} \cos\left(\phi_j - \phi_i\right)\right] + K_0 \sin\left(\varphi_E - \varphi_i + \delta_{Ei}\right).$$

$$\frac{dk_{ij}^E}{dt} = \epsilon\left(K_0\right)\left[\alpha \cos\left(\varphi_i - \varphi_j\right) - k_{ij}\right],$$

$$\frac{dk_{ij}^I}{dt} = \epsilon\left(K_0\right)\left[\alpha \sin\left(\varphi_i - \varphi_j\right) - k_{ij}^I\right].$$

Recapping

- We represent a collection of neurons by the phase of their coherent oscillations.
- The phase difference between stimulus and response oscillators encode a continuum of responses.
- The dynamics comes from inhibitory as well as excitatory neuronal connections.

Outline

Response selection

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Conditional probabilities

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Conditional probabilities

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Outline

What is quantum?

- Nondeterministic.
- Nonlocal.
- Contextual.

Determinism and predictability

- Classical systems can be completely unpredictable (e.g., three-body system, Sinai billiards).
- We cannot distinguish a deterministic from a stochastic dynamics.
- Should we care anyway?

Contextuallity

- Example: $[\hat{P}, \hat{Q}] \neq 0$.
- Not a big deal in social sciences.

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Example:

- Is a cheap date good or bad?
- Rephrasing for this conference: do you think your female friends like cheap dates?
- Did you know dates are on sale at the supermarket?

Nonlocality

- Imagine two parallel sections: Alice and Bob.
- Alice asks supermarket question first.
- Because of Alice's choice, students at Bob's classroom answered yes to the cheap date question.
- Spooky?! Should we care?

What about the brain?

- Stochastic.
- Contextual.
- Nonlocal?

What is quantum in SS? An example

- Should I buy a plot of land given the uncertainties due to the presidential elections?
- If Republican, I decide it is better to buy.
- If Democrat, I also decide it is better to buy.
- Therefore, I should prefer buying over not buying, even if I don't know who will win (Savage's Sure-thing Principle)

- Tversky and Shafir showed that people violate the Sure-thing Principle.

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Oscillator interference

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Some data

- For two stimulus oscillators, s_1 and s_2, and two response oscillators, r_1 and r_2.
- We select couplings between oscillators such that X is selected 60% of the time if s_1 is active, and 50% of the time if s_2 is active.
- By selecting the couplings between s_1 and s_2, we can control the degree of synchronicity between then.
- If s_1 and s_2 are activated, we can have interference between s_1 and s_2.
- In such cases, X is selected less than 40% of the time.

What the \#\$ *! do we know!?

- Propagation of oscillations on the cortex behave like a wave.
- Neural oscillator interference may be sensitive to context.
- Could quantum effects be simply contextual?

Outline

Summary

- A small number of phase oscillators may be used to model a continuum of responses (with results similar to SR theory).
- The model is simple enough such that we can easily understand physically how responses are selected via inhibitory and excitatory couplings.
- Interference may help us understand how complex neural networks have "quantum-like" dynamics.