Response Selection Using Neural Phase Oscillators

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A symposium on the occasion of Patrick Suppes' 90th birthday

March 10, 2012

[1]With Pat Suppes and Gary Oas (Suppes et al., 2012).

Neurons all the way down?

- What scale should we use?
  - Down to the synapse level?
  - Neurons?
  - Collective behavior of neurons?
- For language processing, robustness and measurable macroscopic effects suggest a *large* number of neurons.
- Even for a large collection of neurons, we still have several options with respect to modeling.
  - Do we need detailed interactions between neurons? Are the shapes of the action potential relevant? Timing?
- Our goal is to reduce the number of features, yet retain a physical meaning.

Outline

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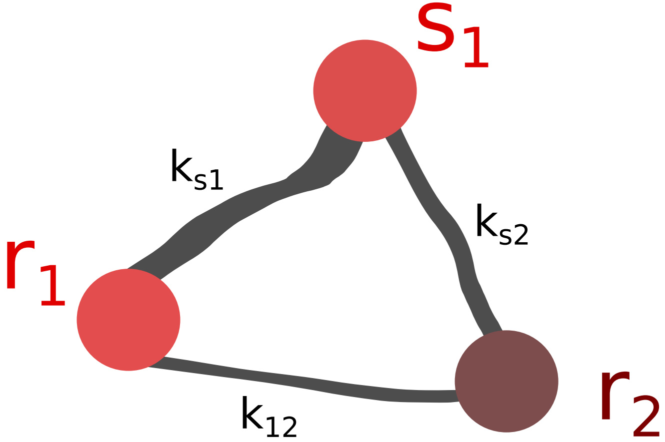
Stimulus and response neurons



How to represent responses with few oscillators?

- Each neural oscillator's dynamics can be described by the phase, φ.

$$s(t) = A_s \cos\varphi_s(t) = A_s \cos(\omega t),$$

$$r_1(t) = A_1 \cos\varphi_{r_1}(t) = A\cos(\omega t + \delta\varphi),$$

$$r_2(t) = A_2 \cos\varphi_{r_2}(t) = A\cos(\omega t + \delta\varphi - \pi).$$

$$I_1 \equiv \left\langle (s(t) + r_1(t))^2 \right\rangle_t = A^2 (1 + \cos(\delta\varphi)).$$

$$I_2 \equiv \left\langle (s(t) + r_2(t))^2 \right\rangle_t = A^2 (1 - \cos(\delta\varphi)).$$

- A response is the balance between the strengths I_1 and I_2,

$$b = \frac{I_1 - I_2}{I_1 + I_2} = \cos(\delta\varphi)$$

Kuramoto Equations

- If no interaction, $\varphi_i = \omega_i t + \delta_i$, and

$$\dot{\varphi}_i = \omega_i .$$

- If we have a weak interaction, then

$$\dot{\varphi}_i = \omega_i - \sum_{j \neq i} A_{ij} \sin\left(\varphi_i - \varphi_j\right).$$

- For fixed phase differences,

$$\dot{\phi}_i = \omega_i + \sum A_{ij} \sin\left(\phi_j - \phi_i + \delta\varphi_{ij}\right).$$

$$\dot{\phi}_i = \omega_i + \sum \left[A_{ij} \sin\left(\phi_j - \phi_i\right) + B_{ij} \cos\left(\phi_j - \phi_i\right)\right].$$

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Reinforcing oscillators

- During reinforcement:

$$\dot{\phi}_i = \omega_i + \sum \left[A_{ij} \sin\left(\phi_j - \phi_i\right) + B_{ij} \cos\left(\phi_j - \phi_i\right)\right] + K_0 \sin\left(\varphi_E - \varphi_i + \delta_{Ei}\right).$$

$$\frac{dk_{ij}^E}{dt} = \epsilon\left(K_0\right)\left[\alpha \cos\left(\varphi_i - \varphi_j\right) - k_{ij}\right],$$

$$\frac{dk_{ij}^I}{dt} = \epsilon\left(K_0\right)\left[\alpha \sin\left(\varphi_i - \varphi_j\right) - k_{ij}^I\right].$$

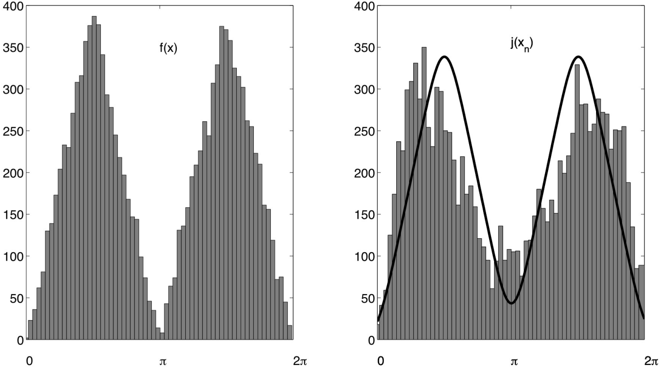
Recapping

- We represent a collection of neurons by the phase of their coherent oscillations.
- The phase difference between stimulus and response oscillators encode a continuum of responses.
- The dynamics comes from inhibitory as well as excitatory neuronal connections.

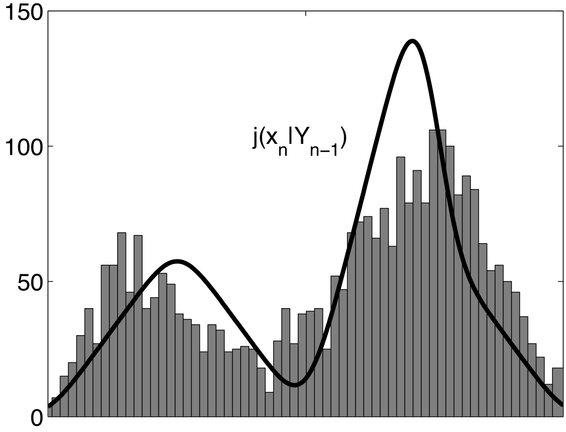
Outline

1. The oscillator model
2. SR theory with neural oscillators
3. Some wild speculations?
4. Summary

Response selection



Conditional probabilities



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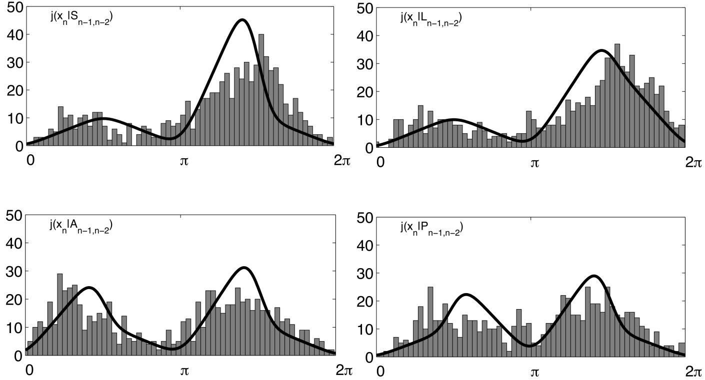

Outline

What the \#\$ *! do we know!?

- Since propagation of oscillations on the cortex behave like a wave, neural oscillator interference may be sensitive to context, like the two slit in physics.
- There are lots of research about whether one could detect "quantum effects" in the brain (see Bruza et al., 2009, and references therein).
- Those quantum effects are not quantum, but contextual (Suppes and de Barros, 2007; de Barros and Suppes, 2009).

Oscillator interference

- Assume we have two stimulus oscillators, s_1 and s_2, and two response oscillators, r_1 and r_2.
- Say oscillators' couplings are such that both s_1 or s_2 select X when activated 60% of the time.
- However, because of oscillator interference, if s_1 and s_2 are activated, X may be selected less than 60% of the time.
- This is similar to the two-slit interference in physics.

Not so wild?!

- Consider the following to be true:
  - If A, then X is preferred over Y.
  - If $\neg A$, then X is preferred over Y.
- Savage's Sure Thing Principle: X should be preferred over Y if we don't know whether A or $\neg A$.
- Shafir and Tversky (1992); Tversky and Shafir (1992) showed that people violate the Sure Thing Principle. So may oscillators.

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Summary

- A small number of phase oscillators may be used to model a continuum of responses (with results similar to SR theory).
- The model is simple enough such that we can easily understand physically how responses are selected via inhibitory and excitatory couplings.
- Interference may help us understand how complex neural networks have "quantum-like" dynamics.

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