Joint probabilities and quantum cognition

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Quantum Interactions Conference

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Quantum cognition

- Many authors discuss quantum-like effects in the social sciences[1], but where do those effects come from?
- We'll try to approach this question by focusing on the brain.

[1] E.g. Haven, E. December 2004 *Physica A: Statistical Mechanics and its Applications* **344(1–2)**, 142–145, Khrennikov, A. and Haven, E. October 2009 *Journal of Mathematical Psychology* **53(5)**, 378–388, Khrennikov, A. (2009) *Biosystems* **95(3)**, 179–187, Khrennikov, A. (2011) *Biosystems* **105(3)**, 250–262.

Outline

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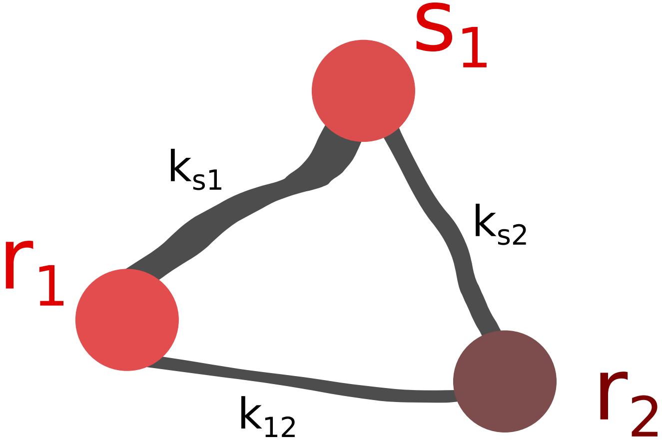
1. A model of response computation
2. Quantum-like behavior
3. Joint probabilities and oscillators
4. Conclusions

Size matters!

- What scale should we use?
  - Down to the synapse level?
  - Neurons?
  - Collective behavior of neurons?
- For language processing, robustness and measurable macroscopic effects suggest a *large* number of neurons.
- Even for a large collection of neurons, we still have several options with respect to modeling.
  - Do we need detailed interactions between neurons? Are the shapes of the action potential relevant? Timing?
- Our goal is to reduce the number of features, yet retain a physical meaning.

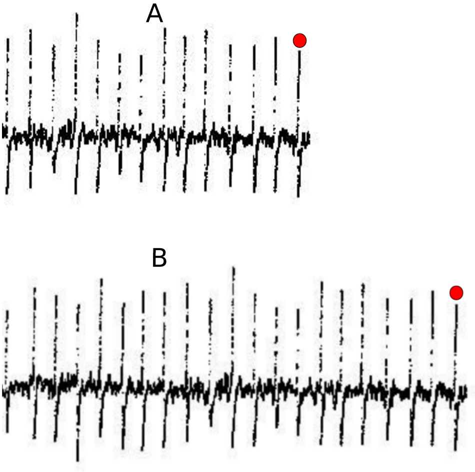
Stimulus and response neurons



[2]

[2] Suppes, P., deBarros, J. A., and Oas, G. April 2012 *Journal of Mathematical Psychology* **56(2)**, 95–117

The intuition



Kuramoto Equations

- If no interaction,

$$O_i\left(t\right)=A_i\cos\varphi_i\left(t\right)=A_s\cos\left(\omega t\right),$$

$$\varphi_i=\omega_i t+\delta_i,$$

and

$$\dot{\varphi}_i=\omega_i.$$

How to represent responses with few oscillators?

- Each neural oscillator's dynamics can be described by the phase, φ.

$$s(t) = A_s \cos \varphi_s(t) = A_s \cos(\omega t),$$

$$r_1(t) = A_1 \cos \varphi_{r_1}(t) = A \cos(\omega t + \delta\varphi),$$

$$r_2(t) = A_2 \cos \varphi_{r_2}(t) = A \cos(\omega t + \delta\varphi - \pi).$$

$$I_1 \equiv \left\langle (s(t) + r_1(t))^2 \right\rangle_t = A^2 (1 + \cos(\delta\varphi)).$$

$$I_2 \equiv \left\langle (s(t) + r_2(t))^2 \right\rangle_t = A^2 (1 - \cos(\delta\varphi)).$$

- A response is the balance between the strengths I_1 and I_2,

$$b = \frac{I_1 - I_2}{I_1 + I_2} = \cos(\delta\varphi)$$

Encoding responses

- To encode responses, we need to modify

$$\dot{\varphi}_i = \omega_i - \sum_{j \neq i} A_{ij} \sin\left(\varphi_i - \varphi_j\right)$$

to include angles, i.e.,

$$\dot{\phi}_i = \omega_i + \sum A_{ij} \sin\left(\phi_j - \phi_i + \delta\varphi_{ij}\right).$$

$$\dot{\phi}_i = \omega_i + \sum \left[A_{ij} \sin\left(\phi_j - \phi_i\right) + B_{ij} \cos\left(\phi_j - \phi_i\right)\right].$$

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Reinforcing oscillators

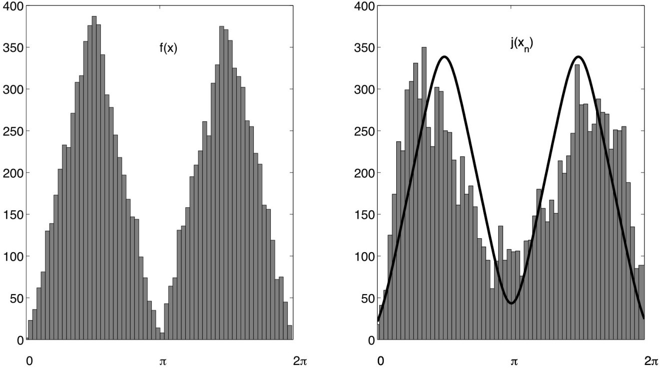
- During reinforcement:

$$\dot{\phi}_i = \omega_i + \sum \left[A_{ij} \sin\left(\phi_j - \phi_i\right) + B_{ij} \cos\left(\phi_j - \phi_i\right)\right] + K_0 \sin\left(\varphi_E - \varphi_i + \delta_{Ei}\right).$$

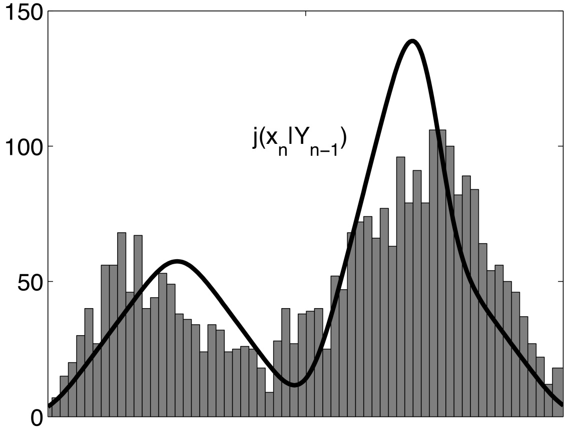
$$\frac{dk_{ij}^E}{dt} = \epsilon\left(K_0\right)\left[\alpha \cos\left(\varphi_i - \varphi_j\right) - k_{ij}\right],$$

$$\frac{dk_{ij}^I}{dt} = \epsilon\left(K_0\right)\left[\alpha \sin\left(\varphi_i - \varphi_j\right) - k_{ij}^I\right].$$

Response selection



Conditional probabilities



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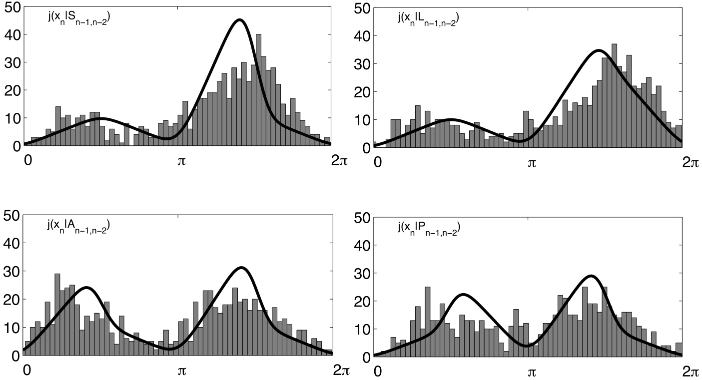

Recapping Part 1

- We represent a collection of neurons by the phase of their coherent oscillations.
- The dynamics comes from inhibitory as well as excitatory neuronal connections.
- The phase difference between stimulus and response oscillators encode a continuum of responses.

Outline

What is quantum?

- Nondeterministic.
- Nonlocal.
- Contextual.

Determinism and predictability

- Classical systems can be completely unpredictable (e.g., three-body system, Sinai billiards).
- We cannot distinguish a deterministic from a stochastic dynamics.
- Should we care anyway?

Contextuallity

- Example: $[\hat{P}, \hat{Q}] \neq 0$.
- Not a big deal in social sciences.

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Example:

- Is a cheap date good or bad?
- Rephrasing for this conference: do you think your female friends like cheap dates?
- Did you know dates are on sale at the supermarket?

Nonlocality

- Imagine two parallel sections: Alice and Bob.
- Alice asks supermarket question first.
- Because of Alice's choice, students at Bob's classroom answered yes to the cheap date question.
- Spooky?! Should we care?

What about the brain?

- Stochastic.
- Contextual.
- Nonlocal?

What is quantum in SS? An example

- Should I buy a plot of land given the uncertainties due to the presidential elections?
- If Republican, I decide it is better to buy.
- If Democrat, I also decide it is better to buy.
- Therefore, I should prefer buying over not buying, even if I don't know who will win (Savage's Sure-thing Principle)

- Tversky and Shafir showed that people violate the Sure-thing Principle.

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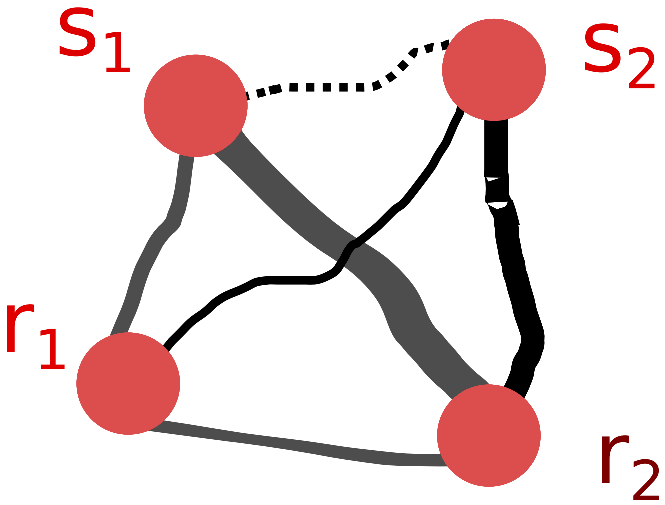
Oscillator interference



Some data

- For two stimulus oscillators, s_1 and s_2, and two response oscillators, r_1 and r_2.
- We select couplings between oscillators such that X is selected 60% of the time if s_1 is active, and 50% of the time if s_2 is active.
- By selecting the couplings between s_1 and s_2, we can control the degree of synchronicity between then.
- If s_1 and s_2 are activated, we can have interference between s_1 and s_2.
- In such cases, X is selected less than 40% of the time.

What the \#\$ *! do we know!?

- Propagation of oscillations on the cortex behave like a wave.
- Neural oscillator interference may be sensitive to context.
- Could quantum effects be simply contextual?

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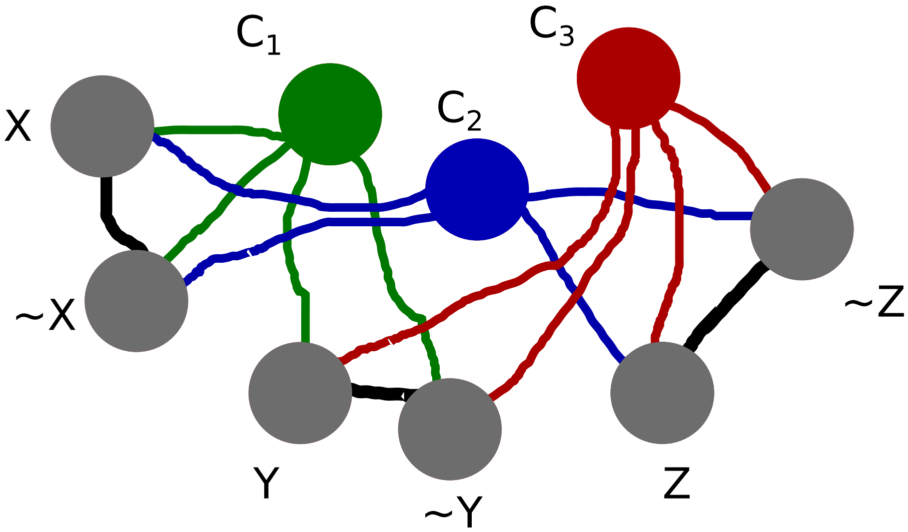
The simplest example

- Let **X**, **Y**, and **Z** be ± 1 random variables with zero expectation.
- Let
$$E(\mathbf{XY}) = E(\mathbf{YZ}) = E(\mathbf{XZ}) = \epsilon.$$
- **X**, **Y**, and **Z** have a joint probability distribution if and only if $\epsilon > -1/3$.
- This is the simplest example of a set of random variables without a joint probability.

Can it make sense?

- It is possible to give a (albeit contrived)example where **X**, **Y**, and **Z** could not have a joint.
- Let each of the correlations, $E(\mathbf{XY}), E(\mathbf{YZ})$, and $E(\mathbf{XZ})$ correspond to different expert opinions which are inconsistent.
- Since the opinions are inconsistent, we don't have a joint.

A not-so-simple oscillator model



But it is not quantum!

- In their quantum version, we would have observables in a Hilbert space corresponding to each random variable **X**, **Y**, and **Z**. Call them $\hat{X}$, $\hat{Y}$, and $\hat{Z}$.
- To say that the correlations $E(\mathbf{XY})$, $E(\mathbf{YZ})$, and $E(\mathbf{XZ})$ have a certain value means that we can observe any pair of **X**, **Y**, and **Z**, i.e. $[\hat{X}, \hat{Y}] = [\hat{X}, \hat{Z}] = [\hat{Z}, \hat{Y}] = 0$.
- But the fact that they commute means we can find a basis where all operators $\hat{X}$, $\hat{Y}$, and $\hat{Z}$ are diagonal.
- Therefore, it is possible to measure simultaneously $\hat{X}$, $\hat{Y}$, and $\hat{Z}$, which means that there exists a joint probability distribution for **X**, **Y**, and **Z**.

Outline

Summary

- A small number of phase oscillators may be used to model a continuum of responses (with results similar to SR theory).
- The model is simple enough such that we can easily understand physically how responses are selected via inhibitory and excitatory couplings.
- Interference may help us understand how complex neural networks have "quantum-like" dynamics.
- Such quantum-like dynamics comes from the contextuallity of oscillators, and are not necessarily compatible with the structure of observables imposed by a quantum Hilbert space.