Quantum gravity correction, evolution
of scalar field and inflation

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Abstract.   We take the first nontrivial coefficient of the Schwinger-DeWitt expansion as a leading correction to the action of the second-derivative metric-dilaton gravity. To fix the ambiguities related with an arbitrary choice of the gauge fixing condition and the parametrization for the quantum field, one has to use the classical equations of motion. As a result, the only corrections are the ones to the potential of the scalar field. It turns out that the parameters of the initial classical action may be chosen in such a way that the potential satisfies most of the conditions for successful inflation.

1   Introduction

The behavior of the scalar field plays a crucial role in most of the inflationary scenarios. In particular, the special type of evolution of the scalar field to a stable vacuum state through the "slow-roll" scheme is in the heart of the inflationary scenario. In order to provide successful inflation, the potential of the scalar field should satisfy some set of conditions, and it is difficult to derive such a potential from a reasonable quantum field theory. According to the modern point of view, inflation takes place at very high energies where the effects of quantum gravity may dominate. Unfortunately, a consistent theory of quantum gravity is unavailable, therefore one has to use some approximate scheme to explore the important metric-scalar models.

   If one supposes that inflation takes place at typical energies below – but not many orders of magnitude below – the Planck scale, then it is possible to use the framework of effective quantum gravity as a forecast for what happens when quantum gravitational effects are taken into account. An interesting aspect in the recent development of the effective approach to quantum gravity [1] (see also [2]) is that the full theory may include many high derivative

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terms, but they are not seen at lower energies when the corresponding heavy particles do not propagate [3] and the contribution of the corresponding loops is suppressed [5]. For this reason, in this work, we disregard the higher derivative effects and concentrate on the quantum contributions to the second-derivative local part of the effective action.

This paper is organized as follows. In the next section we formulate the action and symmetry transformations for the metric-scalar theory in four-dimensional space-time. We choose the theory with the action composed by Einstein-Hilbert action with cosmological constant and by the conformal scalar field. The introduction of the cosmological constant into the classical action is important because, otherwise, one unavoidably meets the cosmological constant in the vacuum state after the quantum corrections are taken into account. As it was discussed in [7, 12, 13], this type of action possesses an extra symmetry which may be called conformal duality. In section 3 we review the previous calculation of the first term in the Schwinger-DeWitt expansion of the one-loop effective action [13], discuss its gauge-fixing dependence and construct the gauge-independent correction to the potential for the scalar field. In section 4 the physical conditions for the effective potential are formulated and the possibility to have successful inflation is explored. In the last section some conclusions and discussions are presented.

2   Metric-dilaton actions and its symmetries.

According to [11, 12], all metric-scalar theories can be divided into two main classes: with and without local conformal symmetry. The models of the first class may be classically equivalent to General Relativity (GR) with cosmological constant, while the second type of models have additional degrees of freedom. All such models are related by a reparametrization of the scalar field supplemented by a conformal transformation of the metric. Therefore, since the conformal symmetry is absent and there is no danger of anomaly, one can safely use any frame for the description of the theory on both classical and quantum levels. Let us write the action of the theory in the form

$$S = S_{B(\phi),\lambda} + S_{L(\phi),\tau},\tag{1}$$

where B, L (and later K) are some smooth functions of ϕ. Both actions $S_{B(\phi),\lambda}$ and $S_{L(\phi),\tau}$ are classically equivalent to GR. This means [12] that

$$S_{B(\phi),\lambda} = \int d^4x \sqrt{-g} \left\{ \frac{3}{2} \frac{B_1^2}{B} g^{\mu\nu} \partial_\mu\phi \, \partial_\nu\phi + B\,R + \lambda\,B^2 \right\},\tag{2}$$

where $B = B(\phi)$ and $B_1 = \frac{dB}{d\phi}$. The last action is invariant under the transformation

$$\bar{\phi} = \bar{\phi}(\phi), \qquad \bar{g}_{\mu\nu} = g_{\mu\nu} \frac{B(\bar{\phi}(\phi))}{B(\phi)}.\tag{3}$$

On the other hand, all such actions (including the case of $B = (16\pi G)^{-1} = const$) may be linked with each other by a conformal transformation of the metric [12]. Also, the models with a non-constant $B(\phi)$ can be linked by a simple reparametrization of the scalar field. As a consequence of this, the theory (1) manifests a conformal duality

$$S\left[g_{\mu\nu}; B(\phi), K(\phi); \lambda, \tau\right] = S\left[\bar{g}_{\mu\nu}; L(\phi), \frac{K(\phi)L(\phi)}{B(\phi)}; \lambda, \tau\right], \qquad \bar{g}_{\mu\nu} = g_{\mu\nu} \frac{L(\phi)}{B(\phi)}.\tag{4}$$

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In a particular case one meets the dual symmetry [7, 12, 13]:

$$\bar{g}_{\mu\nu} \leftrightarrow g_{\mu\nu} = \bar{g}_{\mu\nu} \cdot \frac{\gamma}{B(\phi)}, \qquad \gamma \leftrightarrow \frac{1}{\gamma}, \qquad B(\phi) \leftrightarrow \frac{1}{B(\phi)}, \qquad \lambda \leftrightarrow \tau. \tag{5}$$

This transformation describes the inversion of the coupling constant γ and function $B(\phi)$ and can link the strong and weak coupling regimes. It should be very interesting to apply this symmetry to the study of strong-gravity effects, but in the present paper we will concentrate on the quantum corrections to the classical action (1) and to their influence on classical cosmological models.

3   One-loop calculation and mass-shell conditions

In this section we calculate the one-loop correction to the effective action for the theory with conformal duality and extract its unambiguous part. Since all the theories with conformal duality (1) differ from each other by the reparametrization of the scalar field only, we will perform the calculations for the simplest case

$$S = \frac{1}{\kappa^2} \int d^4x \sqrt{-g} \left\{ \frac{3}{2\phi} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi + (\phi + \gamma)R - V(\phi) \right\}, \tag{6}$$

where we will put later on $\gamma = -1$ and fix $-V(\phi) = \lambda\phi^2 + \tau\gamma^2$.

The quantum calculation can be done within the background field method (see [9] for an introduction). The features of the metric-dilaton theory require the special background gauge, which has been originally introduced in the similar two-dimensional theory [10] and recently applied for the calculation of the one-loop divergences in general four-dimensional metric-scalar theory in [11], and in the case of the theory (6) in [13]. The starting point is the splitting of the fields into background $g_{\mu\nu}, \phi$ and quantum ones, $\bar{h}_{\mu\nu}, h, \varphi$:

$$\phi \to \phi' = \varphi + \kappa\,\phi\,, \qquad g_{\mu\nu} \to g'_{\mu\nu} + \kappa\,h_{\mu\nu}\,, \qquad h_{\mu\nu} = \bar{h}_{\mu\nu} + \frac{1}{4}\,g_{\mu\nu}h\,, \qquad h = h^\mu_\mu\,, \tag{7}$$

where we divided the quantum metric into the trace and the traceless parts. The details of the calculations can be found in [13]. Here we just give a brief review of it. The one-loop correction to the effective action is given by the standard general expression

$$\bar{\Gamma}^{1-loop} = \frac{i}{2}\,\mathrm{Tr}\,\ln\hat{H} - i\,\mathrm{Tr}\,\ln\hat{H}_{ghost} + \frac{i}{2}\,\mathrm{Tr}\,\ln Y^{\mu\nu} \tag{8}$$

where $\hat{H}$ is the Hermitian bilinear form of the action $S + S_{gf}$ with added gauge fixing term

$$(S + S_{gf})^{(2)} = \int d^4x \sqrt{-g} \left(\bar{h}_{\mu\nu},\ h,\ \varphi\right) \left(\hat{H}\right) \left(\bar{h}_{\alpha\beta},\ h,\ \varphi\right)^T$$

where

$$S_{gf} = \int d^4x \sqrt{-g}\,\chi_\mu\,Y^{\mu\nu}\,\chi_\nu\,, \tag{9}$$

$\hat{H}_{ghost}$ is the bilinear form of the action of the Faddeev-Popov ghosts. The general form of the gauge fixing condition and weight function are

$$\chi_\mu = \nabla_\lambda \bar{h}^\lambda_\mu + \beta\,\nabla_\mu h + \rho\,\nabla_\mu\varphi, \qquad\qquad Y^{\mu\nu} = -\alpha\,g^{\mu\nu}\,, \tag{10}$$

where the gauge fixing parameters α, β, ρ are some functions of the background dilaton, which can be fine tuned to simplify the calculations. For instance, if one chooses these functions as

$$\alpha = -\frac{1}{2}\,(\phi + \gamma)\,, \qquad \beta = -\frac{1}{2}, \qquad \rho = -\frac{1}{\phi + \gamma}\,, \qquad (11)$$

then the bilinear forms $\hat{H}$, $\hat{H}_{gh}$ can be reduced to minimal operators. For example[4]

$$\hat{H} = \hat{K}\Box + \hat{L}_\lambda \nabla^\lambda + \hat{M} \qquad (12)$$

and

$$\mathrm{Tr}\,\ln\hat{H} \;=\; \ln\mathrm{Det}\hat{K} \;+\; \mathrm{Tr}\,\ln\left(\hat{1}\Box + \hat{K}^{-1}\hat{L}^\mu \nabla_\mu + \hat{K}^{-1}\hat{M}\right)\,. \qquad (13)$$

The first term gives a simple contribution of an ordinary functional determinant of the c-number matrix. The second term (just as the bilinear operator of the ghost action [13]) has the form of the usual minimal operator

$$\hat{H}_{min} = \hat{1}\Box + \hat{E}^\lambda \nabla_\lambda + \hat{D},$$

to which the Schwinger-DeWitt expansion applies.

Our purpose is to evaluate the quantum gravity corrections to the classical action to the first order in curvature, and in the corresponding second order in the derivatives of the scalar field. This approximation corresponds to the first coefficient of the Schwinger-DeWitt expansion. Thus, for each of the minimal operators we can write

$$\ln\mathrm{Det}\left(-\hat{H}_{min}\right) \;=\; -\,\mathrm{Tr}\,\int_{\epsilon^2}^\infty \frac{ds}{i\,s}\,U_0(x,x';s)\sum_{n=0}^\infty a_n(x,x')\,(i\,s)^n\,, \qquad (14)$$

where the Tr includes taking the usual trace, the coincidence limit $x' = x$ and covariant integration over x. The ultraviolate divergences may be regularized by means of the dimensional parameter ε. However, since we are considering a theory which is not fundamental but is supposed to work below some energy scale, it is natural to introduce the covariant cut-off directly in the integral (14). Then, the correction to the effective action coming from the corresponding operator is nothing but the functional trace of the a_1-coefficient with a factor of some cut-off parameter μ^2, which is related to ε^2.

In this way, taking into account all the terms in (6), we arrive at the following expression

$$\Gamma_1^{(1-loop)} \;=\; \mu^2 \int d^4x\sqrt{-g}\,\left\{\mathcal{A}(\phi)\,g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi + \mathcal{B}(\phi)R + \mathcal{C}(\phi)\right\}\,, \qquad (15)$$

where μ is a dimensional parameter related to ϵ in (14), and

$$\mathcal{A}(\phi) = \frac{1}{(\gamma + \phi)^2}\left[-\frac{1}{4}\frac{\gamma^2}{\phi^2} - 6\frac{\gamma}{\phi} + \frac{23}{2} + \frac{2}{3}\frac{\phi}{\gamma}\right]\,,$$

$$\mathcal{B}(\phi) = -\frac{23}{4} + \frac{2}{3}\frac{\phi}{\gamma}\,, \qquad \mathcal{C}(\phi) = \frac{2}{3}\lambda\phi\left(\frac{5\phi}{\gamma} + 1\right) - \frac{2}{3}\frac{\phi + 15\gamma}{\gamma(\gamma + \phi)}\left(\lambda\phi^2 + \tau\gamma^2\right)\,. \qquad (16)$$

Eq. (15), with the coefficients given in eq. (16), is the first term in the expansion of the one-loop correction to the classical action. Indeed, this expression does not satisfy

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[4]Explicit expressions for the matrices in (12) are given in [13].

the conditions imposed by conformal duality, and therefore in this approximation quantum corrections violate this symmetry. Below we shall analyze the cosmological implications of the classical action (6) with the above quantum corrections.

One has to notice that the expression (15) is strongly ambiguous because all the coefficients (16) depend on the choice of the gauge fixing condition, and on the parametrization of the quantum fields. Thus, before we start to use the above quantum correction in the cosmological framework, it is necessary to extract its unambiguous part. The gauge and parametrization dependence of the effective action is known in general form [14, 15] (see also [9]). For us it is enough to know that this dependence disappears when one uses the one-shell conditions. Hence, in order to extract some invariant quantity from the expression (15), one can assume that the background fields $g_{\mu\nu}$ and ϕ satisfy classical equations of motion. As a consequence, the equation for the scalar field and the trace of the equation for the metric have the form

$$R = 2\gamma\tau\,, \qquad (\nabla\phi)^2 - 2\phi\,(\Box\phi) = \phi^2\,S(\phi)\,, \qquad (17)$$

where $S(\phi) = 4/3\,(\tau\,\gamma - \lambda\,\phi)$ and $(\nabla\phi)^2 = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$. Mathematically, the use of classical equations of motion in the quantum correction is equivalent to a renormalization of both metric and scalar field, which preserves the purely classical form of the kinetic terms for both scalar field and metric. We remark that keeping the classical form of the covariant kinetic term for the metric means removing the quantum correction for the term linear in curvature. Thus, after the corresponding renormalization of metric and scalar, the only (unambiguous) quantum correction is the one to the potential $V(\phi)$.

Technically the problem of going on-shell in the metric-dilaton theory with a non-minimal interaction is not trivial (see, for example, [16, 17]). Here we will follow the considerations of [13]. For any function $F(\phi)$ and for any solution of the equations of motion (17) one can write

$$\int d^4x\sqrt{-g}\,F(\phi)\,\left\{\phi^{-1}\,(\nabla\phi)^2 - 2\,(\Box\phi) - \phi S(\phi)\right\} = 0\,. \qquad (18)$$

Integrating by parts we arrive at

$$\int d^4x\sqrt{-g}\,\left\{\phi^{-1}F(\phi) + 2F_1(\phi)\right\}(\nabla\phi)^2 = \int d^4x\sqrt{-g}\,F(\phi)\,\phi S(\phi)\,. \qquad (19)$$

For any constant C_1 the Eq. (19) is invariant under $F(\phi) \to F(\phi) + C_1/\sqrt{\phi}$. Therefore for any solution of (17) the function $\phi^{-1/2}$ satisfies the condition

$$\int d^4x\sqrt{-g}\,\phi^{-1/2}\,\phi\,S(\phi) = 0\,, \qquad (20)$$

and therefore this term can be always disregarded.

In order to have correspondence with (15), one has to put $\phi^{-1}F(\phi) + 2\,F_1(\phi) = \mathcal{A}(\phi)$. The solution for $F(\phi)$ has the form

$$F(\phi) = C_1\phi^{-1/2} + \phi^{-1/2}\int_{\phi_0}^{\phi}d\phi\,\phi^{1/2}\,\mathcal{A}(\phi)\,, \qquad (21)$$

where the first term can be omitted due to (20). Now (21) must be substituted into (19) and (15). We then arrive at the on-shell quantum correction (see [13] for further details). It is important that the corresponding expression does not depend on the parametrization of the quantum field and on the choice of the gauge parameters.

Before writing down the unambiguous effective potential with the quantum gravity corrections we make some change of notations:

$$\phi = \frac{\chi^2}{12 M_{pl}^2}, \qquad \lambda = -6f M_{pl}^2, \qquad \gamma = 1. \tag{22}$$

In these new variables, the kinetic term for the scalar field acquires the common form $1/2(\nabla \chi)^2$. The effective potential depends on three arbitrary parameters: the energy scale μ, the scalar coupling f, and the dimensionless cosmological constant Λ. In the next section, we will check whether there is a region in the space of parameters μ, Λ, f, in which the physical conditions for V_{eff} to yield suitable inflation are satisfied. Those conditions include, at a first place, the boundness of the potential from below. Then the dynamics of the scalar field will be governed by the form of the potential, and the consistency with the inflationary scenario poses many other restrictions. Thus, despite the fact that we have three parameters in hand, it is not a trivial matter to satisfy all conditions. The next important condition on V_{eff} is that in the ground state the value of the cosmological constant should be very close to zero. The recent estimates show that the observable value of the cosmological constant density $\rho_\Lambda = \Lambda/8\pi G$ is $\rho_\Lambda \approx 0.7\Omega_c \approx (3\text{meV})^4$ where Ω_c is the critical density of the universe. As we shall see, this value is many orders smaller than the typical quantities characterizing our potential. Thus we can safely use the approximation in which the value of the cosmological constant is zero in the ground state χ_0. One has to notice that, in the framework of our model, it is not possible to fine-tune the value of the cosmological constant by adding corresponding "vacuum" term. Such a term should affect the form of the quantum correction in (15). Therefore one has to make a more sophisticated fine tuning changing the value of τ (or Λ) in the starting action. The local conditions for the potential to have zero "physical" cosmological constant in the ground state is

$$V_{eff}(\chi_0, \Lambda, f, \mu) = 0, \qquad \frac{d}{d\chi} V_{eff}(\chi_0, \Lambda, f, \mu) = 0, \tag{23}$$

and these equations for Λ must be solved before one can proceed to the next conditions.

For the subsequent study, it is more adequate to use the adimensional variables:

$$\tau = \Lambda \cdot M_{pl}^2, \qquad \chi = x \cdot M_{pl}, \qquad \mu = y \cdot M_{pl}. \tag{24}$$

We will explore the values of the coupling f ranging from 0 to 1, y is the cutting scale measured in the units of M_{Pl} and x is the dimensionless scalar field also measured in the units of M_{Pl}. The dimensionless cosmological constant Λ will be found in the next section from the condition of vanishing of effective (physical) cosmological constant. In terms of x and y the effective potential reads

$$V_{eff} = \frac{1}{24} f x^4 - \Lambda - y^2 \cdot \left\{ \Lambda \left(15 - \frac{x^2}{9} \right) - \frac{f}{12} x^2 (x^2 - 4) - \frac{4}{3} \frac{x^2 + 180}{x^2 + 12} \left(\Lambda - \frac{f x^4}{24} \right) \right.$$

$$\left. + \frac{8}{9} \left(\Lambda + \frac{f x^2}{2} \right) \cdot \left(1 + \frac{115}{72} x^2 + \frac{17}{2\sqrt{12}} x^2 \arctan \left[\frac{x}{\sqrt{12}} \right] - \frac{199 x^2}{6(x^2 + 12)} \right) \right\}. \tag{25}$$

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4 Conditions for slow-roll inflation

Now we will enumerate the conditions which the potential must satisfy in order to yield a reasonable inflationary scenario [19, 20, 21]. We shall write some of the formulas in terms of the scalar field χ from (22) while others will be written in terms of x.

The slow roll conditions are:

$$\frac{V''}{24\pi V} \ll 1, \tag{26}$$

$$\frac{V'}{\sqrt{48\pi V}} \ll 1. \tag{27}$$

When these conditions are fulfilled, the kinetic term in the energy momentum tensor of the scalar field becomes negligible with respect to the potential term. The energy density and pressure of the field are given by

$$\rho_\chi = \frac{\dot\chi^2}{2} + V, \tag{28}$$

$$p_\chi = \frac{\dot\chi^2}{2} - V, \tag{29}$$

(the gradient terms are damped by expansion), and under these conditions we obtain $p_\chi \approx -\rho_\chi$. This corresponds to a vacuum equation of state, or equivalently, to an effective cosmological constant. The geometry inflates almost exponentially, $a(t) \approx \exp(Ht)$ with H almost constant given by

$$H^2 \approx \frac{8\pi V}{M_P}. \tag{30}$$

When conditions (26, 27) cease to be valid, inflation ends. Hence, they define the initial and final values of the scalar field, x_i and x_f, in whose interval inflation occurs. In order to solve the flatness and isotropy problems, the geometry must inflate the necessary amount in order to bring into causal contact disconnected regions of the early Universe which were found to be thermalized from the observations of the Cosmic Microwave Background Radiation (CMBR), while making them almost flat. This implies that the number of e-folds of growth of the scale factor during this period must be greater then 60 [22]:

$$-8\pi \int_{x_i}^{x_f} \frac{V(x)}{V'(x)}dx = N > 60. \tag{31}$$

Finally, the magnitude of the density perturbations produced in this scenario at the moment they cross outside the horizon must be

$$\left(\frac{\delta\rho}{\rho}\right)_{hor} \approx 10^{-5} \tag{32}$$

in order to not be in conflict with CMBR observations. This is because the anisotropies observed by the COBE satellite in the CMBR [23] are of this order of magnitude and these anisotropies are related with the density perturbations by means of the Sachs-Wolfe effect [24] via the equation

$$\frac{\delta T}{T} \approx \left(\frac{\delta\rho}{\rho}\right)_{hor}, \tag{33}$$

where $\delta T/T$ are the fluctuations in the temperature of the CMBR. This is the most difficult condition which inflaton potentials must satisfy and it reads [22]

$$\frac{\delta\rho}{\rho} \sim -\frac{V^{3/2}(\bar{x})}{M_P^3 V'(\bar{x})} \sim 10^{-5}, \tag{34}$$

where $\bar{x}$ is the value of the scalar field about 50 e-folds *before* the end of inflation, the time where the modes of cosmological interest first cross outside the horizon, which is given by

$$-8\pi \int_{\bar{x}}^{x_f} \frac{V(x)}{V'(x)} dx = 50. \tag{35}$$

We are left with three numeric parameters, f, the coupling strength, Λ, the cosmological constant, and y, representing the energy scale, and the question is for what parameters we have all above conditions satisfied for a suitable inflation.

1. For $y < 0.235$ we can have potentials with the suitable shape and a local minimum that allows slow rolling.

2. For $0.245 > y > 0.235$ we start having other minima and maxima. The potential does not yield slow-roll inflation.

3. For $y > 0.245$ the quantum term dominates for large x and we do not have stable solutions with a global minimum anymore.

We can now use the values $y < 0.235$ in the following way:

1. Fix a value for f;

2. Compute the values of $x_{\min}$ and $\Lambda_{\min}$ such that V_{eff2}, the potential, is zero at its global minimum;

3. Redefine the potential as being V_{final}, which is the effective potential V_{eff2} with Λ changed by $\Lambda_{\min}$;

We now have a potential that has the right form to have inflation, and we need to check whether the conditions are satisfied for different values of f and y. All conditions are satisfied for $y = 0.235$ and $f < 10^{-12}$. Some typical plots exemplifying the three possibilities are shown in Figures $1 - 3$.

5   Conclusion

We have obtained a scalar field potential from the Schwinger-DeWitt expansion of dilaton gravity which is free of the ambiguities related to gauge fixing conditions and which can yield successful inflation for some choices of parameters. It should be interesting to push further the model and calculate the power spectra of scalar and density perturbations, the quadrupole CMBR temperature anisotropies for these two types of perturbations, and compare with observations. Also, it should be interesting to see if the other forms of the potential

having more maxima and minima, which, as we have seen, are possible for other choices of the parameters, can yield inflation but with a $k = -1$ universe.

Acknowledgments

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Figure Captions

Figure 1. This potential shows a possibility for tunneling. The values of parameters are: $f = 5 \cdot 10^{-13}$ and $y = 0.235$. The fine-tuned value of Λ is $\Lambda = -0.719 \cdot 10^{-12}$ and the point of minima of the potential is $x_0 = 11.43$.

Figure 2. This potential fulfills all the conditions for the slow-roll inflation (without tunelling). The values for the parameters are: $y = 0.229$ and $f = 10^{-14}$. Fine-tuned "bare" Λ is $\Lambda = -0.439 \cdot 10^{-15}$ and the value of x where the potential has a minimum is $x_0 = 1.423$. Inequality (26) is fulfilled with it being always less than 0.001. Inequality (27) is satisfied from $x = 0.0001$ to $x = 0.6$. In this interval the number of e-folds is 99.95, satisfying (31). Equation (35) implies that $\bar{x} = 0.0085$. In this case, the left hand side of (34) takes the approximate value of $7 \cdot 10^{-5}$. Obviously, the desirable behavior is provided by the very small value of coupling f (which, when reduced, can make the lhs of (34) even smaller) and consequent smoothness of the potential. Quantum corrections are also small and do not change the asymptotic behavior of the potential at $x \to \infty$.

Figure 3. Potential with dominating quantum corrections, which are not bounded from below. This type of potentials one always meets for y greater than 0.244.