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Bohmian trajectories for an evaporating blackhole

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Abstract

In this work we apply Bohm's interpretation to the quantized spherically-symmetric blackhole coupled to a massless scalar field. We show that the quantum trajectories for linear combinations of eigenstates of the Wheeler–DeWitt equation form a large set of different curves that cannot be predicted by the standard interpretation of quantum mechanics. Some of them are consistent with the expected value of the time derivative of the mass, whereas other trajectories are not, because they represent blackholes that switch from absorbing to emitting regimes.
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Since the fundamental discovery, made by Hawking, that blackholes may emit radiation [1], many studies have been made in order to better understand this process. Initially, most of the works were concentrated in the area of quantum field theory in curved space–time [2]. More recently, some physicists have started studying Hawking's radiation with the aid of a quantum gravity theory [3–6]. Most of these works deal with the theory of quantum general relativity. In this theory, the standard probabilistic Copenhagen interpretation of quantum mechanics cannot be applied. The use of different interpretations of quantum mechanics have been proposed to deal with quantum general relativity [7–11], and among them is the causal or de Broglie–Bohm interpretation.

The causal interpretation of quantum mechanics was first proposed by de Broglie, and later on it was extended by Bohm to include many-particle systems and fields [12]. In this interpretation, variables corresponding to observable physical quantities have an ontological meaning regardless of whether they are observed or not, contrary to the standard Copenhagen interpretation of quantum mechanics. The problems of applying Copenhagen's interpretation of quantum mechanics to quantum cosmology has raised recent interest on the causal interpretation of quantum

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mechanics in quantum cosmology [13,14], as this interpretation does not need an external observer to bring a observable into reality. The causal interpretation has been applied with success, by several authors, to quantum general relativity [13–19].

In the present work we would like to continue our previous study [20] on the Hawking radiation process using the theory of quantum general relativity and the causal interpretation. We shall use, once more, the two wave-functionals derived in Tomimatsu's work [3]. The first wave-functional (Ψ_c; see Eq. (1)), was interpreted by Tomimatsu as representing the classical blackhole behavior, mainly because the expected value of the time derivative of the mass of the blackhole is positive. The second wave-functional (Ψ_q, Eq. (2)), was interpreted as representing the quantum blackhole behavior, as the expected value of the time derivative of the blackhole mass is negative. Furthermore, the mass loss rate is in agreement with the one derived directly from the Hawking emission process [21].

In our earlier work [20], we showed that one may obtain evolution equations for the blackhole mass when one considers quantum states described either by Ψ_c or Ψ_q. This result is relevant because from the standard interpretation of quantum mechanics used in Tomimatsu's work one has just expected values for the time derivative of the blackhole mass, whereas the quantum trajectories for the evaporating blackholes yield a change in the rate of emission consistent with earlier results [21]. In this work we will extend this result to include states that will have different behavior from the ones obtained by Tomimatsu.

In this Letter, first, we compute Bohmian trajectories for several different cases and show the existence of states that may, during some time, behave quantum mechanically, emitting Hawking radiation, and, some other time, behave classically, absorbing energy. This is done by studying sates described by linear combinations of Ψ_c and Ψ_q. Each wave-functional is characterized by two parameters: one present in the phase and the other in the modulus. Therefore, one may have a state which is the result of the linear combination of two different Ψ_c. As we shall see, the mass evolution trajectories for this particular combination may have a very different behavior from the one found in [20] for a state represented by a single Ψ_c. We shall also consider the other possible linear combinations: Ψ_c with Ψ_q and Ψ_q with Ψ_q. We must emphasize that the fluctuating behavior of those superpositions cannot be obtained from the standard interpretation of quantum mechanics, as it would not show up in the expectations for the rate of change of mass. Also, when this behavior appears, the WKB approximation cannot always be used, as the effects of the quantum potential, and therefore the rate of change of the absolute value of the wave functional, may be strong.

From [3] we have the following solutions to the Wheeler–DeWitt equation

$$\Psi_c = C \exp\left[i\left(\frac{R}{4} - \frac{k^2}{2R} + k\Phi\right)\right], \tag{1}$$

and

$$\Psi_q = C \exp\left[i\left(\frac{R}{4} + \frac{k^2}{2R}\right) - |k\Phi|\right], \tag{2}$$

where k and C are arbitrary real and complex parameters, respectively, and R and Φ are the physical fields [20]. Tomimatsu argued that Ψ_c represents the classical blackhole behavior [3]. One way to understand Tomimatusu's argument is the following. If we impose the classical constraint [20] $H = \sqrt{2}(P_\Phi^2/2R^2 - P_R + 1/4)$, with the aid of the expression for the canonical moment $P_R = \dot{M}/2 + 1/4$, we obtain at once that $\dot{M} \geqslant 0$. It means that the apparent horizon increases and the blackhole absorbs. This is expected, as we have an ingoing null fluid with positive energy density. Quantum mechanically, we can see this same behavior if we use Ψ_c and compute the expectation value of $\dot{M}$, $\langle\dot{M}\rangle$, where over-dot means a derivative with respect to the advanced time $v \equiv t + r$ [3], finding positive value equal to $\frac{k^2}{4M^2}$. Also, the scalar field sector is described in (1) by scalar waves penetrating the apparent horizon from the exterior region.

On the other hand, Ψ_q in Eq. (2) represents the quantum-mechanical blackhole behavior [3], as in this case the value of $\langle\dot{M}\rangle$ is given by $\langle\dot{M}\rangle = -\frac{k^2}{4M^2}$. This value is always negative, which means that the apparent horizon decreases and the blackhole emits. The scalar field cannot penetrate the horizon, as it is exponentially suppressed. This can be interpreted as a classically forbidden state.

Now, let us see what the causal interpretation tell us about the states described by Ψ_c (1) and Ψ_q (2). Following the causal interpretation formalism applied to quantum general relativity [14], if we write our wave-functionals as

$$\Psi = \mathcal{R}\exp(iS), \tag{3}$$

we obtain dynamical equations for the physical variables from

$$P_{X_i} = \frac{\delta S}{\delta X_i}, \tag{4}$$

where X_i stands for R and Φ. The expression for the quantum potential Q is given, in the present situation by

$$Q = -\frac{\nabla^2\mathcal{R}}{\mathcal{R}}. \tag{5}$$

We shall restrict our attention to R, as the equations for Φ are trivial. Starting with Ψ_c, we may write (4), for $X_i = R$, as

$$P_R = \frac{1}{4} + \frac{k^2}{2R^2}. \tag{6}$$

Now, introducing the expression of P_R, Eq. (15) from Ref. [20], we obtain

$$\dot{M} = \frac{k^2}{4M^2}. \tag{7}$$

This equation is easily integrated to give

$$M^3 = \frac{3}{4}k^2(v - v_0) + M_0^3, \tag{8}$$

where v_0 and M_0 are the initial values of v and M, respectively. This solution tells us that the blackhole mass M increases continuously as the time, measured by v, increases. This wave-functional is associated with the classical behavior of the blackhole. In particular, if we compute the value of the quantum potential Q from Eq. (5) for Ψ_c in Eq. (1), we find that it is zero, as expected for the classical situation.

Similarly, for Ψ_q, we find the dynamical equation for M,

$$\dot{M} = -\frac{k^2}{4M^2}. \tag{9}$$

Note that Eq. (9) is similar to the equation for the expected value of $\dot{M}$. The difference is that Eq. (9) can be integrated to give the exact evolution of M and not just its expectation giving

$$M^3 = -\frac{3}{4}k^2(v - v_0) + M_0^3, \tag{10}$$

where v_0 and M_0 are the initial values of v and M, respectively. Eq.(10) tell us that if the blackhole has an initial mass M_0 at v_0 after a time $v_e = 4M_0^3/3k^2 + v_0$, it will completely evaporate. This is in accordance with the qualitative predictions made by Hawking that, taking into account quantum properties, blackholes evaporate [1]. Eq. (10) is also in accordance with predictions on how this evaporation should take place, if one considers the *elementary particle* picture of blackhole emission [21]. The quantum potential Q in Eq. (5), computed for Ψ_q in Eq. (2), is given by $-k^2/2R^2$. This may be interpreted as an attractive potential that pulls R to zero. If we take into account that $R = 2M$ in Tomimatsu's gauge [3], this potential also pulls the mass towards zero.

Even though the cases presented above are interesting, as they go beyond expected values and predict that the rate of change in the mass of an evaporating blackhole decreases as the mass increase, they still do not give us

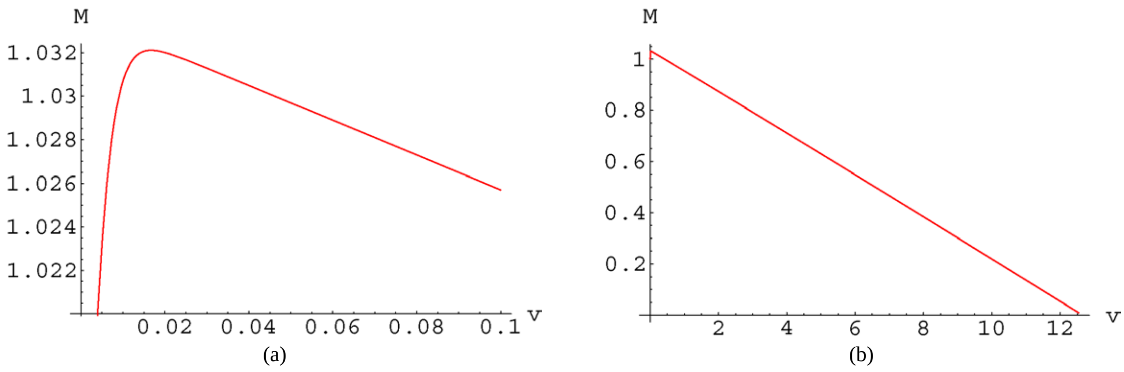

Fig. 1. M versus v for a state described by Ψ_{cc} (11) with $C_{c1}=1$, $C_{c2}=1/2$, $k_{c1}=1$, $k_{c2}=10$, $M(v=0)=1$ and $\Phi(v=0)=0.1$. (a) Small values of v, (b) greater values of v.

any physics that could not be obtained from using a WKB approximation. However, let us see what the causal interpretation tells us about the states described by linear combinations of the wave-functionals in Eqs. (1) and (2).

We start with a general linear combination of two "classical" states Ψ_c,

$$\Psi_{cc}=C_{c1}\exp\left[i\left(\frac{R}{4}-\frac{k_{c1}^2}{2R}+k_{c1}\Phi\right)\right]+C_{c2}\exp\left[i\left(\frac{R}{4}-\frac{k_{c2}^2}{2R}+k_{c2}\Phi\right)\right], \tag{11}$$

where k_{c1}, k_{c2}, C_{c1}, and C_{c2} are constants. Following the steps shown above, we obtain from Ψ_{cc}

$$\begin{aligned}\dot{M}(v)=&\frac{2C_{c1}C_{c2}\cos(\beta_1)(k_{c1}^2+k_{c2}^2+4M(v)^2)}{8(C_{c1}^2+C_{c2}^2+2C_{c1}C_{c2}\cos(\beta_1))M(v)^2}\\&+\frac{C_{c1}^2(2k_{c1}^2+4M(v)^2)+C_{c2}^2(2k_{c2}^2+4M(v)^2)}{8(C_{c1}^2+C_{c2}^2+2C_{c1}C_{c2}\cos(\beta_1))M(v)^2}-\frac{1}{2},\end{aligned} \tag{12}$$

$$\dot{\Phi}(v)=\frac{1}{4M(v)^2}\frac{C_{c1}^2k_{c1}+C_{c2}^2k_{c2}+C_{c1}C_{c2}(k_{c1}+k_{c2})\cos(\beta_1)}{C_{c1}^2+C_{c2}^2+2C_{c1}C_{c2}\cos(\beta_1)}, \tag{13}$$

where

$$\beta_1=\frac{(k_{c1}-k_{c2})(k_{c1}+k_{c2}-4M(v)\Phi(v))}{4M(v)}.$$

This system of coupled differential equations can be integrated numerically for a set of initial conditions and parameters. Performing numerical investigations on a large number of different initial conditions, we see that most of them behaves as expected, i.e., from an initial value M_0 the mass increases as v increases. For some initial conditions, though, we notice a behavior quite different from the expected one, as we can find solutions where the mass initially increase from M_0 for increasing v, reaching a maximum and decreasing afterwards. We stress that when M is small, the oscillations of the wave make the WKB approximation unsuitable, and Bohm's interpretation gives new results. Fig. 1 shows a typical trajectory for the latter case.

Consider now, the general linear combination of two Ψ_q given by

$$\Psi_{qq}=C_{q1}\exp\left[i\left(\frac{R}{4}+\frac{k_{q1}^2}{2R}\right)-|k_{q1}\Phi|\right]+C_{q2}\exp\left[i\left(\frac{R}{4}+\frac{k_{q2}^2}{2R}\right)-|k_{q2}\Phi|\right], \tag{14}$$

where k_{q1}, k_{q2}, C_{q1}, and C_{q2} are constants. Following the same steps as before, we obtain

$$\dot{M}(v) = \frac{-e^{(k_{q1}+k_{q2})\Phi(v)} C_{q2} \cos\left(\frac{k_{q1}^2 - k_{q2}^2}{4M(v)}\right)(k_{q1}^2 + k_{q2}^2 - 4M(v)^2)}{4D_1(M,\Phi)M(v)^2} - \frac{e^{2k_{q2}\Phi(v)} C_{q2}^2 (2k_{q2}^2 - 4M(v)^2)}{4D_1(M,\Phi)M(v)^2} + \frac{e^{2k_{q1}\Phi(v)}(-2k_{q1}^2 + 4M(v)^2)}{8D_1(M,\Phi)M(v)^2} - \frac{1}{2}, \tag{15}$$

$$\dot{\Phi}(v) = \frac{1}{4M(v)^2} \frac{e^{(k_{q1}+k_{q2})\Phi(v)}(k_{q1} - k_{q2}) C_{q2} \sin\left(\frac{k_{q1}^2 - k_{q2}^2}{4M(v)}\right)}{D_1(M,\Phi)}, \tag{16}$$

where, without loss of generality, we set $C_{q1} = 1$, and where we used the abbreviation

$$D_1(M,\Phi) = \left(e^{2k_{q1}\Phi(v)} + e^{2k_{q2}\Phi(v)} C_{q2}^2 + 2e^{(k_{q1}+k_{q2})\Phi(v)} C_{q2} \cos\left(\frac{k_{q1}^2 - k_{q2}^2}{4M(v)}\right)\right).$$

Eqs. (15) and (16) can be integrated numerically. However, we can easily prove that the rhs of (15) is always negative, and as a consequence, the mass always decrease. In Fig. 2, we see a typical mass trajectory for this case.

Finally, let us consider the general linear combination of a "classical" state Ψ_c with a quantum state Ψ_q,

$$\Psi_{cq} = C_c \exp\left[i\left(\frac{R}{4} - \frac{k_c^2}{2R} + k_c\Phi\right)\right] + C_q \exp\left[i\left(\frac{R}{4} + \frac{k_q^2}{2R}\right) - |k_q\Phi|\right], \tag{17}$$

where k_c, k_q, C_c, and C_q are constants. We obtain at once

$$\dot{M}(v) = \frac{-2e^{\sqrt{k_q^2\Phi(t)^2}} C_q \cos(\beta_2)(-k_c^2 + k_q^2 - 4M(v)^2)}{8\left(e^{2\sqrt{k_q^2\Phi(v)^2}} + C_q^2 + 2e^{\sqrt{k_q^2\Phi(v)^2}} C_q \cos(\beta_2)\right)M(v)^2} + \frac{e^{2\sqrt{k_q^2\Phi(v)^2}}(2k_c^2 + 4M(v)^2) + C_q^2(-2k_q^2 + 4M(v)^2)}{8\left(e^{2\sqrt{k_q^2\Phi(v)^2}} + C_q^2 + 2e^{\sqrt{k_q^2\Phi(v)^2}} C_q \cos(\beta_2)\right)M(v)^2} - \frac{1}{2}, \tag{18}$$

$$\dot{\Phi}(v) = \frac{1}{4M(t)^2} \frac{e^{\sqrt{k_q^2\Phi(v)^2}}\left(e^{\sqrt{k_q^2\Phi(v)^2}} k_c \sqrt{k_q^2\Phi(v)^2} + k_c C_q \cos(\beta_2)\sqrt{k_q^2\Phi(v)^2}\right)}{\left(e^{2\sqrt{k_q^2\Phi(v)^2}} + C_q^2 + 2e^{\sqrt{k_q^2\Phi(v)^2}} C_q \cos(\beta_2)\right)\sqrt{k_q^2\Phi(v)^2}} - \frac{1}{4M(t)^2} \frac{e^{\sqrt{k_q^2\Phi(v)^2}} k_q^2 C_q \Phi(v) \sin(\beta_2)}{\left(e^{2\sqrt{k_q^2\Phi(v)^2}} + C_q^2 + 2e^{\sqrt{k_q^2\Phi(v)^2}} C_q \cos(\beta_2)\right)\sqrt{k_q^2\Phi(v)^2}}, \tag{19}$$

where, without loss of generality, we set $C_c = 1$, and use the abbreviation

$$\beta_2 = \frac{k_c^2 + k_q^2 - 4k_c M(v)\Phi(v)}{4M(v)}.$$

Integrating (18)–(19) numerically on a large number of trajectories for M, depending on the value of the fraction k_c/k_q, we found several different types of trajectories. When $k_c/k_q \gg 1$, most of the mass trajectories behave as if they were a superposition of two states Ψ_c. This behavior shows that, in this case, the classical component dominates the quantum component. In the case that $k_c/k_q \ll 1$, most trajectories behave as if they were a quantum-quantum superposition, showing the domination of the quantum component for this case. Finally, in the case that $k_c/k_q \sim 1$, we have trajectories that mix the classical and quantum behaviors. In Fig. 3, we may see an example of

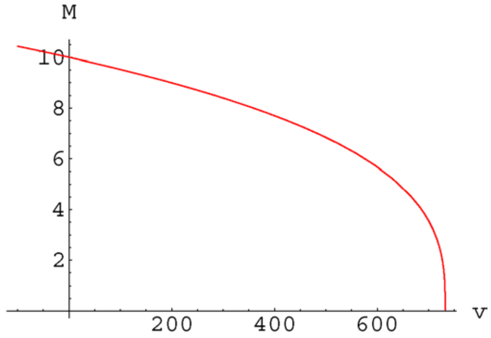

Fig. 2. M versus v for a state described by Ψ_{qq} (14) with $C_q = 1$, $k_{q1} = 1$, $k_{q2} = 2$, $M(v = 0) = 10$ and $\Phi(v = 0) = -1$.

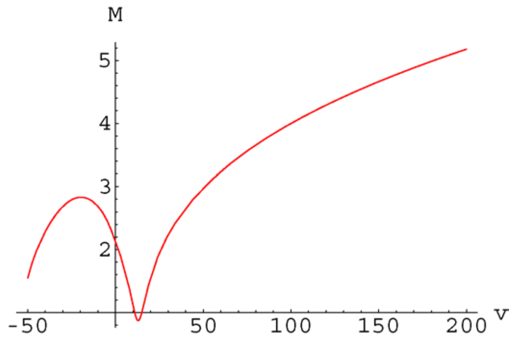

Fig. 3. M versus v for a state described by Ψ_{cq} (17) with $C_q = 1$, $k_c = 1$, $k_q = 2$, $M(v = 0) = 2.135$ and $\Phi(v = 0) = 0.0410$.

such trajectories. The mass increases initially from a given initial value, up to a maximum, and then decreases to a local minimum, increasing thereafter.

It is important to notice that several of the trajectories for the mass computed above cannot be predicted by the expected value of $\dot{M}$. This is because the expected value is a fixed number for each quantum state considered, and does not say anything about the behavior of individual systems in the ensemble.

To summarize, using the causal interpretation, we computed the individual quantum trajectories determined by the initial conditions. We showed that the quantum trajectories for the blackhole mass could either increase or decrease with time, depending on the wave-functions, Ψ_c or Ψ_q. We also showed that for superpositions of those wave-functions new behavior is predicted that cannot be obtained from the standard interpretations of quantum mechanics. In particular, for some states and initial conditions, it was possible to have a blackhole that would start absorbing mass and then, after some time, start evaporating. In some cases the evaporation would last a short amount of time and the absorption process would restart, but in some other cases all the mass evaporates.

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