Tunneling probability for the birth of an asymptotically de Sitter universe

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In the present work, we quantize a closed Friedmann-Robertson-Walker model in the presence of a positive cosmological constant and radiation. It gives rise to a Wheeler-DeWitt equation for the scale factor which has the form of a Schrödinger equation for a potential with a barrier. We solve it numerically and determine the tunneling probability for the birth of a asymptotically DeSitter, inflationary universe, initially, as a function of the mean energy of the initial wave function. Then, we verify that the tunneling probability increases with the cosmological constant, for a fixed value of the mean energy of the initial wave function. Our treatment of the problem is more general than previous ones, based on the WKB approximation. That is the case because we take into account the fact that the scale factor (a) cannot be smaller than zero. It means that, one has to introduce an infinity potential wall at $a = 0$, which forces any wave packet to be zero there. That condition introduces new results, in comparison with previous works.

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I. INTRODUCTION

Since the pioneering work in quantum cosmology due to DeWitt [1], many physicists have worked in this theory. The main motivation behind quantum cosmology is a consistent explanation for the origin of our Universe. So far, the most appealing explanation is the spontaneous *creation from nothing* [2–7]. In that picture for the origin of the Universe, the Universe is a quantum mechanical system with zero size. There is a potential barrier that the Universe may tunnel with a well-defined, nonzero probability. If the Universe actually tunnels, it emerges to the right of the barrier with a definite size. The application of the *creation from nothing* idea in minisuperspace models has led to several important results. The wave function of the Universe satisfies the Wheeler-DeWitt equation [1,8]. Therefore, one needs to specify boundary conditions in order to solve that equation and find a unique and well-defined wave function. The motivation to obtain a wave function that represents the *creation from nothing* has led to the introduction of at least three proposals for the boundary conditions for the wave function of the Universe [7]. The inflationary period of the Universe appears very naturally from the *creation from nothing* idea. That is the case because most of the minisuperspace models considered so far have a potential that decreases, without a limit, to the right of the barrier. It gives rise to a period of unbounded expansion, which is interpreted as the inflationary period of the Universe [7]. Also, it was shown by several authors that an open inflationary Universe may be created from nothing, in theories of a single scalar field for generic potentials [9–11]. Another important issue is the particle content in the Universe originated during the *creation from nothing* process [6,12,13].

In the present work, we would like to explicitly compute the quantum mechanical probability that the Universe tunnels through a potential barrier and initiates its classical evolution. That probability is the tunneling probability (TP) and the particular model we consider here is a closed Friedmann-Robertson-Walker (FRW) model in the presence of a positive cosmological constant and radiation. The radiation is treated by means of the variational formalism developed by Schutz [14]. That model has already been treated quantum mechanically using the ADM formalism and the Dirac quantization for constrained systems [7,15,16]. The wave function, for that model, was calculated in the WKB approximation. Here, we compute the wave function and TP exactly, without any approximation. It will be done by means of a numerical calculation. In particular, our treatment of the problem is more general than previous ones, because we take into account the fact that the scale factor (a) cannot be smaller than zero. It means that, one has to introduce an infinity potential wall at $a = 0$, which forces any wave packet to be zero there. As

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we shall see, that condition introduces new results, in comparison with previous works. This model has two free parameters: the radiation energy and the cosmological constant. Therefore, we will obtain the TP as a function of those two parameters. One of the main motivations of any quantum cosmological model is to fix initial conditions for the classical evolution of our Universe [2]. Here, for the present model, we would gain information on what is the most probable amount of radiation in the initial evolution of the classical Universe and the most probable value of the cosmological constant. Another motivation of the present work, is trying to contribute to a long standing debate about which is the most appropriate set of initial conditions for the wave function of the Universe. The most well-known proposals for the set of initial conditions are the *tunneling* one, due to A. Vilenkin [3], and the *no-boundary* one, due to J. B. Hartle and S. W. Hawking [4]. The application of those proposals for simple models showed that they give some different predictions for the initial evolution of the Universe [3,4,13,15–18]. One of such predictions, which we shall explore here, is the initial energy of the Universe right after its nucleation. The *tunneling* wave function predicts that the Universe must nucleate with the largest possible vacuum energy whereas the *no-boundary* wave function predicts just the opposite [7]. In terms of our results, if one assumes that the cosmological constant describes a vacuum energy, it is important to see if TP increases or decreases with the cosmological constant, for fixed radiation energy. The first behavior favors the *tunneling* wave function and the latter favors the *no-boundary* wave function.

In the next Section, we describe the classical dynamics of the present cosmological model. We write the super-Hamiltonian constraint and the Hamilton's equations. We solve the Hamilton's equations and find the general solution of the model. Then, we qualitatively describe all possible classical evolutions. In Sec. III, we canonically quantize the model and obtain the corresponding Wheeler-DeWitt equation. We solve it, numerically, for particular values of the radiation energy and the cosmological constant. We show the square modulus of the wave function of the Universe as a function of the scalar factor. The tunneling process can be readily seen from that figure. Section IV is divided in three subsections with the main results of the paper. In Subsec. IV A, we start introducing the tunneling probability, then we evaluate its dependence on the radiation energy. We obtain that the TP increases with the radiation energy for a fixed cosmological constant. Therefore, it is more probable that the classical evolution should start with the greatest possible value for the radiation energy. In Subsec. IV B, we give a detailed comparison between the exact TP, computed in the previous subsection, and the corresponding WKB tunneling probability. Here, we show how the presence of an infinity potential wall at $a = 0$ may lead to a difference between our results and previous ones, based on the *WKB* approximation. In the final Subsec. IV C of this section, we evaluate the dependence of the exact TP with the cosmological constant. We obtain that, the TP increases with the cosmological constant for a fixed radiation energy. Therefore, it is more probable that the classical evolution should start with the greatest possible value for the cosmological constant. This behavior of *TP* also favors the *tunneling* wave function. Finally, in Sec. V we summarize the main points and results of our paper.

II. THE CLASSICAL MODEL

The Friedmann-Robertson-Walker cosmological models are characterized by the scale factor $a(t)$ and have the following line element:

$$ds^2 = -N^2(t)dt^2 + a^2(t)\left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2\right), \quad (1)$$

where $d\Omega^2$ is the line element of the two-dimensional sphere with unitary radius, $N(t)$ is the lapse function and k gives the type of constant curvature of the spatial sections. Here, we are considering the case with positive curvature $k = 1$ and we are using the natural unit system, where $\hbar = c = G = 1$. The matter content of the model is represented by a perfect fluid with four-velocity $U^\mu = \delta^\mu_0$ in the comoving coordinate system used, plus a positive cosmological constant. The total energy-momentum tensor is given by

$$T_{\mu\nu} = (\rho + p)U_\mu U_\nu - p g_{\mu\nu} - \Lambda g_{\mu\nu}, \quad (2)$$

where ρ and p are the energy density and pressure of the fluid, respectively. Here, we assume that $p = \rho/3$, which is the equation of state for radiation. This choice may be considered as a first approximation to treat the matter content of the early Universe and it was made as a matter of simplicity. It is clear that a more complete treatment should describe the radiation, present in the primordial Universe, in terms of the electromagnetic field.

Einstein's equations for the metric (1) and the energy-momentum tensor (2) are equivalent to the Hamilton equations generated by the super-Hamiltonian constraint

$$\mathcal{H} = -\frac{p_a^2}{12a} - 3a + \Lambda a^3 + \frac{p_T}{a}, \quad (3)$$

where p_a and p_T are the momenta canonically conjugated to a and T the latter being the canonical variable associated with the fluid [19]. The total Hamiltonian is given by $N\mathcal{H}$ and we shall work in the conformal gauge, where $N = a$.

The classical dynamics is governed by the Hamilton equations, derived from the total Hamiltonian $N\mathcal{H}$, namely

$$\begin{cases} \dot{a} = \frac{\partial(N\mathcal{H})}{\partial p_a} = -\frac{p_a}{6}, \\ \dot{p}_a = -\frac{\partial(N\mathcal{H})}{\partial a} = 6a - 4\Lambda a^3, \\ \dot{T} = \frac{\partial(N\mathcal{H})}{\partial p_T} = 1, \\ \dot{p}_T = -\frac{\partial(N\mathcal{H})}{\partial T} = 0. \end{cases} \tag{4}$$

Where the dot means derivative with respect to the conformal time $\tau \equiv Nt$. We also have the constraint equation $\mathcal{H} = 0$. We have the following solutions for the system (4):

$$T(\tau) = \tau + c_1,$$
$$a(\tau) = \frac{\sqrt{6\beta}}{\sqrt{3 + \sqrt{9 - 12\Lambda\beta}}} \times sn\left(\frac{\sqrt{18 + 6\sqrt{9 - 12\Lambda\beta}}(\tau - \tau_0)}{6}, \sigma\right), \tag{5}$$

where c_1, β and τ_0 are integration constants, sn is the Jacobi's elliptic sine [20] of modulus σ given by

$$\sigma = \frac{\sqrt{2}}{2}\sqrt{\frac{-2\beta\Lambda + 3 - \sqrt{9 - 12\Lambda\beta}}{\Lambda\beta}}. \tag{6}$$

In the case studied here $\Lambda > 0$, the radiation energy β can take values in the domain, $\beta \leq 3/(4\Lambda)$. If one substitutes values of β such that $\beta < 3/(4\Lambda)$ in Eqs. (5) and (6), the scale factor, starting from zero, expands to a maximum size and then recollapse. On the other hand, if $\beta = 3/(4\Lambda)$, the scalar factor initially decelerates and then, enter the regime of unbounded expansion.

III. THE QUANTUM MODEL

We wish to quantize the model following the Dirac formalism for quantizing constrained systems [21]. First we introduce a wave function which is a function of the canonical variables $\hat{a}$ and $\hat{T}$

$$\Psi = \Psi(\hat{a}, \hat{T}). \tag{7}$$

Then, we impose the appropriate commutators between the operators $\hat{a}$ and $\hat{T}$ and their conjugate momenta $\hat{P}_a$ and $\hat{P}_T$. Working in the Schrödinger picture, the operators $\hat{a}$ and $\hat{T}$ are simply multiplication operators, while their conjugate momenta are represented by the differential operators

$$p_a \to -i\frac{\partial}{\partial a}, \qquad p_T \to -i\frac{\partial}{\partial T}. \tag{8}$$

Finally, we demand that the operator corresponding to $N\mathcal{H}$ annihilate the wave function Ψ, which leads to Wheeler-DeWitt equation

$$\left(\frac{1}{12}\frac{\partial^2}{\partial a^2} - 3a^2 + \Lambda a^4\right)\Psi(a, \tau) = -i\frac{\partial}{\partial \tau}\Psi(a, \tau), \tag{9}$$

where the new variable $\tau = -T$ has been introduced.

The operator $N\hat{\mathcal{H}}$ is self-adjoint [22] with respect to the internal product

$$(\Psi, \Phi) = \int_0^\infty da \Psi(a, \tau)^* \Phi(a, \tau), \tag{10}$$

if the wave functions are restricted to the set of those satisfying either $\Psi(0, \tau) = 0$ or $\Psi'(0, \tau) = 0$, where the prime $'$ means the partial derivative with respect to a. Here, we consider wave functions satisfying the former type of boundary condition and we also demand that they vanish when a goes to ∞.

The Wheeler-DeWitt Eq. (9) is a Schrödinger equation for a potential with a barrier. We solve it numerically using a finite difference procedure based on the Crank-Nicholson method [23,24] and implemented in the program GNU-OCTAVE. Our choice of the Crank-Nicholson method is based on its recognized stability. The norm conservation is commonly used as a criterion to evaluate the reliability of the numerical calculations of the time evolution of wave functions. In Refs. [25,26], this criterion is used to show analytically that the Crank-Nicholson method is unconditionally stable. Here, in order to evaluate the reliability of our algorithm, we have numerically calculated the norm of the wave packet for different times. The results thus obtained show that the norm is preserved.

In fact, numerically one can only treat the *tunneling from something* process, where one gives an initial wave function with a given mean energy, very concentrated in a region next to $a = 0$. That initial condition fixes an energy for the radiation and the initial region where a may take values. Our choice for the initial wave function is the following normalized gaussian,

$$\Psi(a, 0) = \left(\frac{8192E^3}{\pi}\right)^{1/4} a e^{(-4Ea^2)}, \tag{11}$$

where E is the radiation energy. $\Psi(a, 0)$ is normalized by demanding that the integral of $|\Psi(a, 0)|^2$ from 0 to ∞ be equal to one and its mean energy be E. After one gives the initial wave function, it propagates following the appropriate Schrödinger equation until it reaches infinity in the a direction. Numerically, one has to fix the infinity at a finite value. In the present case we fix $a_{\max} = 30$ as the infinity in the a direction. The general behavior of the solutions, when E is smaller than the maximum value of the potential barrier, is an everywhere well-defined, finite, normalized wave packet. Even in the limit when the scale factor goes to zero. For small values of a the wave packet have great amplitudes and oscillates rapidly due to the interaction between the incident and reflected components. The transmitted component is an oscillatory wave packet that moves to the right and has a decreasing amplitude which goes to

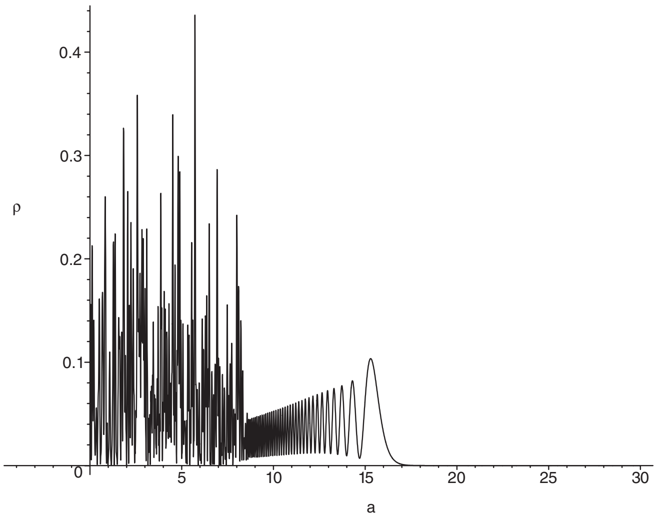

FIG. 1. $|\Psi(a, t_{\max})|^2 \equiv \rho$, for $\Lambda = 0.0121$, $E = 185$ at the moment $t_{\max}$ when Ψ reaches infinity, located at $a = 30$.

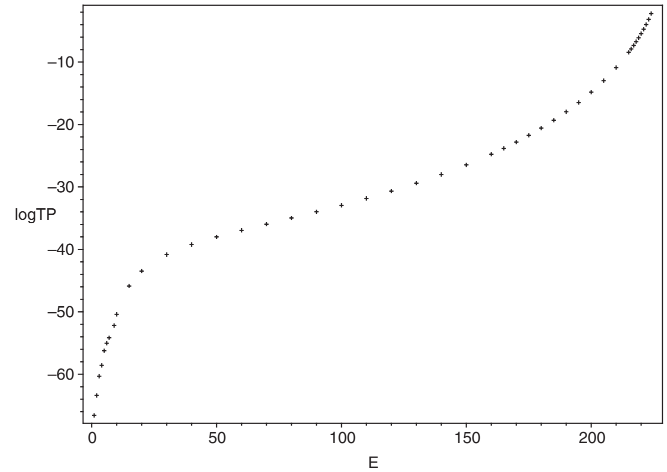

FIG. 2. $\log TP_{\rm int}$ for different radiation energies (E) for a fixed $\Lambda = 0.01$.

zero in the limit when a goes to infinity. As an example, we solve Eq. (9) with $\Lambda = 0.0121$. For this choice of Λ the potential barrier has its maximum value equal to 185.95. In order to see the tunneling process, we choose $E = 185$ for the initial wave function Eq. (11). For that energy, we compute the points where it meets the potential barrier, the left (a_{ltp}) and right (a_{rtp}) turning points. They are, $a_{ltp} = 10.7287$ and $a_{rtp} = 11.5252$. In Fig. 1, we show $|\Psi(a, t_{\max})|^2$ for the values of Λ and E, given above, at the moment ($t_{\max}$) when Ψ reaches infinity. For more data on this particular case see Table III in Appendix A. It is important to mention that the choice of the numerical values for Λ and E above and in the following examples, in the next section, were made simply for a better visualization of the different properties of the system.

IV. TUNNELING PROBABILITIES

A. Tunneling probability as a function of *E*

We compute the tunneling probability as the probability to find the scale factor of the Universe to the right of the potential barrier. In the present situation, this definition is given by the following expression:

$$TP_{\rm int} = \frac{\int_{a_{rtp}}^{\infty} |\Psi(a, t\max)|^2 da}{\int_0^{\infty} |\Psi(a, t\max)|^2 da}, \tag{12}$$

where, as we have mentioned above, numerically ∞ has to be fixed to a maximum value of a. Here, we are working with $a_{\max} = 30$.

Since, by normalization, the denominator of Eq. (12) is equal to the identity, $TP_{\rm int}$ is effectively given by the numerator of Eq. (12). We consider initially the dependence of TP on the energy E. Therefore, we compute $TP_{\rm int}$ for many different values of E for a fixed Λ. For all cases, we consider the situation where E is smaller than the maximum value of the potential barrier. From that numerical study we conclude that the tunneling probability grows with E for a fixed Λ. As an example, we consider 47 values of the radiation energy for a fixed $\Lambda = 0.01$. For this choice of Λ the potential barrier has its maximum value equal to 225. In order to study the tunneling process, we fix the mean energies of the various $\Psi(a, 0)$'s Eq. (11) to be smaller than that value. In Table II in Appendix A, we can see, among other quantities, the different values of the energy E, $TP_{\rm int}$, a_{ltp} and a_{rtp} for each energy. In Fig. 2, we see the tunneling probability as functions of E, for this particular example. Because of the small values of some TP's, we plot the logarithms of the TP's against E.

Since TP grows with E it is more likely for the Universe, described by the present model, to nucleate with the highest possible radiation energy. Therefore, it is more probable that the classical evolution should start with the greatest possible value for the radiation energy.

B. Exact tunneling probability versus WKB tunneling probability

Let us, now, compare the exact tunneling probability represented by $TP_{\rm int}$ Eq. (12) with the approximated WKB tunneling probability ($TP_{\rm WKB}$). The $TP_{\rm WKB}$ is defined by the ratio between the square modulus of the transmitted amplitude of the WKB wave function and the square modulus of the incident amplitude of the WKB wave function [27,28]. For the present situation, it is given by the following expression [28],

$$TP_{\rm WKB} = \frac{4}{(2\theta + \frac{1}{(2\theta)})^2}, \tag{13}$$

where,

$$\theta = \exp\left(\int_{a_{ltp}}^{a_{rtp}} da\sqrt{12(3a^2 - \Lambda a^4 - E)}\right). \qquad (14)$$

It is important to note that the $TP_{\rm WKB}$ Eq. (13), was computed considering that the incident wave (Ψ_I) reaches the potential barrier at a_{ltp}. Then, part of Ψ_I is transmitted to ∞ (Ψ_T) and part is reflected to $-\infty$ (Ψ_R). In the present problem, we have an infinity potential wall at $a = 0$ because the scale factor cannot be smaller than zero. It means that Ψ_R cannot go to $-\infty$, as was assumed in order to compute the $TP_{\rm WKB}$. Instead, Ψ_R will reach the infinity potential wall at $a = 0$ and will be entirely reflected back toward the potential barrier, giving rise to a new incident wave ($(\Psi_R)_I$). The new incident wave $(\Psi_R)_I$ reaches the potential barrier at a_{ltp} and is divided in two components. A reflected component which moves toward the infinity potential wall at $a = 0$ ($((\Psi_R)_I)_R$) and a transmitted component which moves toward ∞ ($((\Psi_R)_I)_T$). $((\Psi_R)_I)_T$ will contribute a new amount to the already existing $TP_{\rm int}$ due to (Ψ_T). On the other hand, $((\Psi_R)_I)_R$, after being reflected at $a = 0$, gives rise to a new incident wave which will contribute a further amount to the already existing $TP_{\rm int}$. If we let our system evolve for a long period of time, $TP_{\rm int}$ will get many such contributions from the different reflected components. The only way it makes sense comparing $TP_{\rm int}$ with $TP_{\rm WKB}$ is when we let the system evolve for a period of time (Δt) during which Ψ_R cannot be reflected at $a = 0$ and come back to reach the potential barrier. It is clear by the shape of our potential that the greater the mean energy E of the wave packet (11), the greater is (Δt). As an example, Table I in Appendix A, shows a comparison between $TP_{\rm int}$ and $TP_{\rm WKB}$ for different values of E and Δt for the case with $\Lambda = 0.01$. We can see, clearly, that both tunneling probabilities coincide if we consider the appropriate Δt, for each E.

In order to have an idea of how the $TP_{\rm int}$ may differ from the $TP_{\rm WKB}$, we let the initial wave packet (11), with different mean energies, evolve during the same time interval Δt. We consider the example given in the previous subsection IVA, with a common time interval of 100. Observing Table I, we see that this amount of Δt guarantees that Ψ_R of the wave packet with mean energy 223 does not contribute to the $TP_{\rm int}$. Therefore, we may expect that to all wave packets with mean energies smaller than 223, $TP_{\rm int}$ will be greater than $TP_{\rm WKB}$. We show this comparison in Table II in Appendix A, where we have an entry for $TP_{\rm WKB}$. It means that, we computed the values of the $TP_{\rm WKB}$s for each energy used to compute the $TP_{\rm int}$s, in the case where $\Lambda = 0.01$. In Fig. 3, we show, graphically, that comparison between both tunneling probabilities as functions of E, with $\Delta t = 100$ for all values of E. Because of the small values of some TP's, we plot the logarithms of the TP's against E.

As we can see from Fig. 3, for this choice of Δt the tunneling probabilities disagree for most values of E. They only agree for values of E very close to the top of the potential barrier. There, because the values of E are similar to 223, Δt is almost sufficient to guarantee that Ψ_R of each wave packet does not contribute to the $TP_{\rm int}$.

As we have mentioned above, numerically one can only treat the *tunneling from something* process, where one gives a initial wave function with a given mean energy, very concentrated in a region next to $a = 0$. Then, if we take $E = 0$ the $TP_{\rm int}$ will be zero. On the other hand, we may have an idea how $TP_{\rm int}$ behaves near $E = 0$ from Fig. 2. From Table II in Appendix A, one can see that $TP_{\rm WKB} = 7.0246 \times 10^{-522}$ when $E = 0$.

Finally, we may compute the time (τ) the Universe would take, for each energy, to nucleate. In order to understand the meaning of τ, consider a photon that composes the radiation which is initially confined to the left of the potential barrier. Then, compute the emission probability of that photon, per unit of time. We may invert it to obtain τ, the time the photon would take to escape the potential barrier. If we consider τ as the time it takes for the most part of the photons to escape the barrier, we obtain the time the Universe would take, for each energy, to appear at the right of the barrier. In the present situation, τ is given by the following expression [27],

$$\tau = 2a_{ltp}\frac{1}{PT_{\rm int}} \qquad (15)$$

From Table II in Appendix A, we may see the values of τ, for each energy. It is clear by the results that, the smaller the energy E the longer it will take for the Universe to nucleate.

C. Tunneling probability as a function of Λ

We would like to study, now, how the tunneling probability depends on the cosmological constant. In order to do

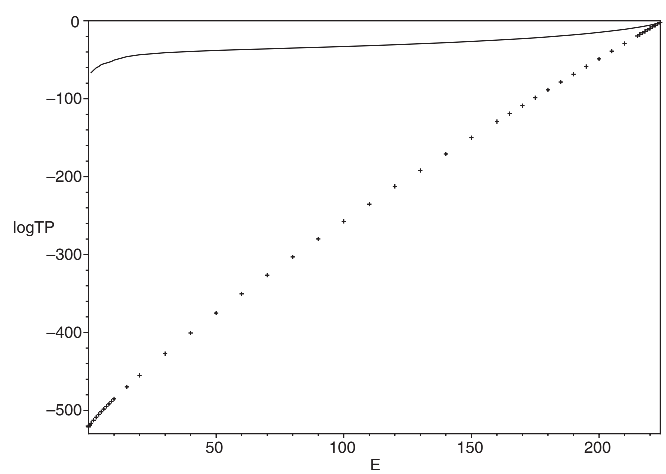

FIG. 3. Comparison between $\log TP_{\rm WKB}$ (dots) and $\log TP_{\rm int}$ (line) for different radiation energies (E) for a fixed $\Lambda = 0.01$. The integration time Δt is equal to 100 for all values of E.

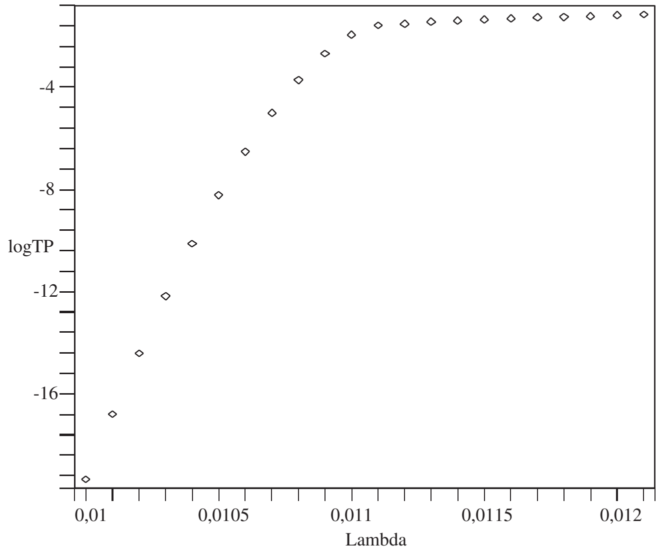

FIG. 4. $\log TP_{\text{int}}$ for 22 different values of Λ for a fixed $E = 185$.

that, we must fix an initial energy E for the radiation and compute the TP_{int} for various values of Λ. For all cases, we consider the situation where E is smaller than the maximum value of the potential barrier. From that numerical study we conclude that the tunneling probability grows with Λ for a fixed E. As an example, we choose $E = 185$ and 22 different values of Λ, such that, the maximum energy of the potential barrier (PE_{max}), for each Λ, is greater than 185. The values of Λ, TP_{int}, PE_{max}, τ, a_{ltp} and a_{rtp} are given in Table III in Appendix A. With those values, we construct the curve TP_{int} versus Λ, shown in Fig. 4. Because of the small values of some TP's, we plot the logarithms of the TP's against Λ.

Since TP grows with Λ it is more likely for the Universe, described by the present model, to nucleate with the highest possible cosmological constant. Therefore, it is more probable that the classical evolution should start with the greatest possible value for the cosmological constant. Also, if we assume that Λ describes a vacuum energy, this result is qualitatively in accordance with the prediction of the *tunneling* wave function due to A. Vilenkin [3].

V. CONCLUSIONS

In the present work, we canonically quantized a closed Friedmann-Robertson-Walker (FRW) model in the presence of a positive cosmological constant and radiation. The radiation was treated by means of the variational formalism developed by Schutz [14]. The appropriate Wheeler-DeWitt equation for the scale factor has the form of a Schrödinger equation for a potential with a barrier. We solved it, numerically, and determined the tunneling probability for the birth of an asymptotically DeSitter, inflationary Universe, as a function of the radiation energy and the cosmological constant. In particular, our treatment of the problem is more general than previous ones, because we took into account the fact that the scale factor (a) cannot be smaller than zero. It means that one has to introduce an infinity potential wall at $a = 0$, which forces any wave packet to be zero there. As we saw, that condition introduced new results, in comparison with previous works. Then we verified that the tunneling probability increases with the radiation energy for a fixed cosmological constant. Therefore, it is more probable that the classical evolution should start with the greatest possible value for the radiation energy. We also gave a detailed comparison between the exact TP, computed here, and the corresponding WKB tunneling probability. Finally, we evaluated the dependence of the exact TP with the cosmological constant. We obtained that the TP increases with the cosmological constant for a fixed radiation energy. Therefore, it is more probable that the classical evolution should start with the greatest possible value for the cosmological constant. Also, if one assumes that the cosmological constant describes a vacuum energy, the latter result seems to be in accordance with the predictions of the *tunneling* wave function [3].

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APPENDIX A: TABLES

TABLE I. A comparison between TP_{int} and TP_{WKB} for 10 different values of E with its associated integration time Δt for the case with $\Lambda = 0.01$.

E	TP_{int}	TP_{WKB}	Δt
80	6.0648×10^{-302}	1.1845×10^{-303}	13
100	1.5887×10^{-259}	3.7271×10^{-258}	18.5
130	6.4375×10^{-194}	9.8051×10^{-193}	30
160	6.9194×10^{-130}	5.7774×10^{-130}	45.5
175	6.5061×10^{-100}	2.1361×10^{-99}	54.5
190	5.3119×10^{-69}	2.6372×10^{-69}	65.5
200	2.2682×10^{-49}	1.7295×10^{-49}	73.5
215	5.4531×10^{-20}	4.1983×10^{-20}	88
219	5.2168×10^{-12}	2.4754×10^{-12}	93
223	7.0045×10^{-04}	1.3731×10^{-04}	100

TABLE II. The computed values of $TP_{\rm int}$, $TP_{\rm WKB}$, τ, a_{ltp} and a_{rtp} for 47 different values of E when $\Lambda = 0.01$.

Energia	$TP_{\rm int}$	$TP_{\rm WKB}$	τ	$altp$	$artp$
0.0000	0.0000	7.0246×10^{-522}	∞	0.0000	17.3205
1.0000	2.5795×10^{-67}	2.7574×10^{-517}	$4.4790 \times 10^{+66}$	0.5777	17.3109
2.0000	3.9975×10^{-64}	2.7181×10^{-513}	$4.0896 \times 10^{+63}$	0.8174	17.3012
3.0000	4.8040×10^{-61}	1.5939×10^{-509}	$4.1702 \times 10^{+60}$	1.0017	17.2915
4.0000	2.6388×10^{-59}	6.6774×10^{-506}	$8.7714 \times 10^{+58}$	1.1573	17.2818
5.0000	5.7738×10^{-57}	2.1799×10^{-502}	$4.4844 \times 10^{+56}$	1.2946	17.2721
6.0000	9.3459×10^{-56}	5.8369×10^{-499}	$3.0366 \times 10^{+55}$	1.4190	17.2623
7.0000	6.9178×10^{-55}	1.3258×10^{-495}	$4.4337 \times 10^{+54}$	1.5335	17.2525
8.0000	7.0061×10^{-56}	2.6169×10^{-492}	$4.6827 \times 10^{+55}$	1.6404	17.2427
9.0000	6.3878×10^{-53}	4.5691×10^{-489}	$5.4506 \times 10^{+52}$	1.7409	17.2328
10.0000	3.9939×10^{-51}	7.1563×10^{-486}	$9.1944 \times 10^{+50}$	1.8361	17.2229
15.0000	1.3310×10^{-46}	1.8319×10^{-470}	$3.3888 \times 10^{+46}$	2.2553	17.1731
20.0000	3.3918×10^{-44}	9.0816×10^{-456}	$1.5401 \times 10^{+44}$	2.6119	17.1224
30.0000	1.4814×10^{-41}	7.5933×10^{-428}	$4.3450 \times 10^{+41}$	3.2183	17.0189
40.0000	5.8991×10^{-40}	2.5466×10^{-401}	$1.2679 \times 10^{+40}$	3.7397	16.9120
50.0000	9.8017×10^{-39}	8.0358×10^{-376}	$8.5875 \times 10^{+38}$	4.2086	16.8014
60.0000	1.1252×10^{-37}	3.9314×10^{-351}	$8.2507 \times 10^{+37}$	4.6419	16.6869
70.0000	1.1121×10^{-36}	4.1409×10^{-327}	$9.0821 \times 10^{+36}$	5.0499	16.5680
80.0000	1.0627×10^{-35}	1.1845×10^{-303}	$1.0236 \times 10^{+36}$	5.4391	16.4443
90.0000	1.0557×10^{-34}	1.0939×10^{-280}	$1.1016 \times 10^{+35}$	5.8147	16.3153
100.0000	1.1488×10^{-33}	3.7271×10^{-258}	$1.0760 \times 10^{+34}$	6.1803	16.1803
110.0000	1.4319×10^{-32}	5.2113×10^{-236}	$9.1338 \times 10^{+32}$	6.5393	16.0386
120.0000	2.1333×10^{-31}	3.2602×10^{-213}	$6.4634 \times 10^{+31}$	6.8942	15.8893
130.0000	3.9754×10^{-30}	9.8051×10^{-193}	$3.6464 \times 10^{+30}$	7.2479	15.7311
140.0000	9.7584×10^{-29}	1.5060×10^{-171}	$1.5582 \times 10^{+29}$	7.6029	15.5626
150.0000	3.3597×10^{-27}	1.2439×10^{-150}	$4.7398 \times 10^{+27}$	7.9623	15.3819
160.0000	1.7562×10^{-25}	5.7774×10^{-130}	$9.4858 \times 10^{+25}$	8.3293	15.1863
165.0000	1.5321×10^{-24}	1.0157×10^{-119}	$1.1118 \times 10^{+25}$	8.5171	15.0818
170.0000	1.5472×10^{-23}	1.5685×10^{-109}	$1.1259 \times 10^{+24}$	8.7085	14.9720
175.0000	1.8431×10^{-22}	2.1361×10^{-99}	$9.6624 \times 10^{+22}$	8.9045	14.8563
180.0000	2.6520×10^{-21}	2.5758×10^{-89}	$6.8673 \times 10^{+21}$	9.1059	14.7337
185.0000	4.7418×10^{-20}	2.7600×10^{-79}	$3.9286 \times 10^{+20}$	9.3142	14.6029
190.0000	1.0919×10^{-18}	2.6372×10^{-69}	$1.7457 \times 10^{+19}$	9.5310	14.4624
195.0000	3.3916×10^{-17}	2.2544×10^{-59}	$5.7545 \times 10^{+17}$	9.7585	14.3099
200.0000	1.5114×10^{-15}	1.7295×10^{-49}	$1.3233 \times 10^{+16}$	10.0000	14.1421
205.0000	1.0542×10^{-13}	1.1943×10^{-39}	$1.9466 \times 10^{+14}$	10.2605	13.9543
210.0000	1.3129×10^{-11}	7.4432×10^{-30}	$1.6069 \times 10^{+12}$	10.5485	13.7379
215.0000	3.6494×10^{-09}	4.1983×10^{-20}	$5.9628 \times 10^{+09}$	10.8801	13.4767
216.0000	1.2796×10^{-08}	3.7003×10^{-18}	$1.7121 \times 10^{+09}$	10.9545	13.4164
217.0000	4.7368×10^{-08}	3.2487×10^{-16}	$4.6582 \times 10^{+08}$	11.0325	13.3523
218.0000	1.8642×10^{-07}	2.8413×10^{-14}	$1.1926 \times 10^{+08}$	11.1150	13.2837
219.0000	7.8683×10^{-07}	2.4754×10^{-12}	$2.8476 \times 10^{+07}$	11.2029	13.2097
220.0000	3.6052×10^{-06}	2.1485×10^{-10}	$6.2674 \times 10^{+06}$	11.2978	13.1286
221.0000	1.8228×10^{-05}	1.8577×10^{-8}	$1.2512 \times 10^{+06}$	11.4018	13.0384
222.0000	1.0419×10^{-04}	1.6002×10^{-6}	$2.2110 \times 10^{+05}$	11.5187	12.9352
223.0000	7.0045×10^{-04}	1.3731×10^{-4}	$3.3281 \times 10^{+04}$	11.6558	12.8118
224.0000	5.9816×10^{-03}	1.1671×10^{-2}	$3.9562 \times 10^{+03}$	11.8322	12.6491

TABLE III. The computed values of $TP_{\rm int}$, $PE_{\rm max}$, τ, a_{ltp} and a_{rtp} for 22 different values of Λ when $E = 185$.

Λ	$TP_{\rm int}$	$PE_{\rm max}$	τ	a_{ltp}	a_{rtp}
0.0100	4.7449×10^{-20}	$2.2500 \times 10^{+02}$	$3.9260 \times 10^{+20}$	9.3142	14.6029
0.0101	1.9386×10^{-17}	$2.2277 \times 10^{+02}$	$9.6427 \times 10^{+17}$	9.3467	14.4800
0.0102	5.5162×10^{-15}	$2.2059 \times 10^{+02}$	$3.4010 \times 10^{+15}$	9.3803	14.3571
0.0103	1.0768×10^{-12}	$2.1845 \times 10^{+02}$	$1.7487 \times 10^{+13}$	9.4152	14.2343
0.0104	1.4239×10^{-10}	$2.1635 \times 10^{+02}$	$1.3276 \times 10^{+11}$	9.4515	14.1114
0.0105	1.2522×10^{-08}	$2.1429 \times 10^{+02}$	$1.5156 \times 10^{+09}$	9.4892	13.9882
0.0106	7.1354×10^{-07}	$2.1226 \times 10^{+02}$	$2.6708 \times 10^{+07}$	9.5286	13.8645
0.0107	2.5363×10^{-05}	$2.1028 \times 10^{+02}$	$7.5462 \times 10^{+05}$	9.5697	13.7402
0.0108	5.3391×10^{-04}	$2.0833 \times 10^{+02}$	$3.6009 \times 10^{+04}$	9.6129	13.6151
0.0109	6.1795×10^{-03}	$2.0642 \times 10^{+02}$	$3.1259 \times 10^{+03}$	9.6583	13.4888
0.0110	3.5077×10^{-02}	$2.0455 \times 10^{+02}$	$5.5342 \times 10^{+02}$	9.7062	13.3610
0.0111	8.4175×10^{-02}	$2.0270 \times 10^{+02}$	$2.3183 \times 10^{+02}$	9.7570	13.2314
0.0112	9.5984×10^{-02}	$2.0089 \times 10^{+02}$	$2.0443 \times 10^{+02}$	9.8112	13.0996
0.0113	1.2079×10^{-01}	$1.9912 \times 10^{+02}$	$1.6341 \times 10^{+02}$	9.8692	12.9648
0.0114	1.3117×10^{-01}	$1.9737 \times 10^{+02}$	$1.5143 \times 10^{+02}$	9.9318	12.8264
0.0115	1.4639×10^{-01}	$1.9565 \times 10^{+02}$	$1.3662 \times 10^{+02}$	10.0000	12.6834
0.0116	1.6190×10^{-01}	$1.9397 \times 10^{+02}$	$1.2446 \times 10^{+02}$	10.0752	12.5344
0.0117	1.7538×10^{-01}	$1.9231 \times 10^{+02}$	$1.1586 \times 10^{+02}$	10.1594	12.3773
0.0118	1.8752×10^{-01}	$1.9068 \times 10^{+02}$	$1.0938 \times 10^{+02}$	10.2559	12.2088
0.0119	1.9940×10^{-01}	$1.8908 \times 10^{+02}$	$1.0402 \times 10^{+02}$	10.3703	12.0232
0.0120	2.1463×10^{-01}	$1.8750 \times 10^{+02}$	$9.7983 \times 10^{+01}$	10.5150	11.8082
0.0121	2.2964×10^{-01}	$1.8595 \times 10^{+02}$	$9.3439 \times 10^{+01}$	10.7287	11.5252

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