Chapter 1
Can quantum non-identity exist in social phenomena?

J. Acacio de Barros

Abstract In this chapter, I discuss indistinguishability and identity in quantum mechanics. I then argue that, similarly to quantum particles, digital money has indistinguishable aspects. This indistinguishability suggests that one can apply quantum-like statistical mechanics to financial systems where digital money is used.

I met Professor Andrei Khrennikov for a brief chat in 2007 during the first Quantum Interaction conference at Stanford University. In 2012 Andrei invited me to give a talk at his legendary annual conference on the foundations of quantum mechanics, to which I became a regular participant. During those conferences (and others), I had a better opportunity to interact with Andrei. I was struck by his enthusiasm about quantum physics and quantum-like social sciences. So much so that he (together with Emmanuel Haven, who co-edited this book with Arkady Plotnitsky) motivated me to continue investigating the connection between quantum probability and the social sciences. I believe this chapter fits the spirit of Andrei's work on quantum-like phenomena. It is a privilege to dedicate this paper to him and have it as part of a Festschrift in his honor.

1.1 Introduction

Quantum mechanics has many peculiar characteristics that are hard to reconcile with a classical view of the world. For instance, quantum properties are contextual because one cannot consistently assign properties without reference to a measurement process (Kochen and Specker, 1967). Since a property depends on an observer's measurement, this aspect has been used to argue that specific interpretations of quantum mechanics are idealistic. What "something is" may depend on the choices made by an observer.

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Another peculiarity of the quantum world is nonlocality (Einstein et al., 1935). Nonlocality is a form of contextuality-at-a-distance, where the properties of an entangled quantum system seem to be instantly connected to those of another far-away system. This connection is not explainable by properties set at a common source in the system's past lightcone (Bell, 1964, 1966).

However, perhaps among the strangest aspects of quantum theory, the indistinguishability of objects is arguably one of the most puzzling. It is a property of quantum objects that, for certain circumstances, they are absolutely indistinguishable in a most fundamental way. Schrödinger, for instance, argues that indistinguishability goes beyond the purely epistemic aspects implied by the word, i.e., that we cannot distinguish between two particles because we lack information about them Schrödinger (1950). Instead, he argues that quantum particles are indistinguishable because they lack identity. They do not seem to have an explicit ontology that can be thought about in classical terms. This lack of identity seems to have broad consequences in our description of the quantum world (de Barros et al., 2017b, 2019; de Barros and Holik, 2020; de Barros et al., 2021).

Recent research has also suggested that some strange aspects of quantum physics have equivalents in the social sciences. For example, researchers have used quantum-like formulations to model order effects in decision-making (Wang et al., 2014)[1], contextuality in semantics (Aerts, 2009), and pricing options (Haven, 2002), to name a few applications. The application of quantum formalisms to the social sciences has become a burgeoning area, with many scientific papers and books published.

Though it is unclear why the quantum formalism adequately describes social phenomena, some researchers proposed plausible explanations for this effectiveness. For example, in decision-making, the contextuality of outcomes could be the consequence of a neural-oscillator Stimulus-Response-like model of how the brain works (de Barros, 2012b). In this neural-oscillator network, the monotonicity of classical probability theory breaks down because of wave-like interference between oscillators. Another possibility could be that human decision-makers use intuitionistic logic. If this is the case, then the quantum algebra of orthomodular quantum lattices approximates the intuitionistic lattice (Narens, 2014). Finally, quantum-like phenomena may be simply a manifestation of contextual observations (de Barros and Suppes, 2009). In this situation, quantum mechanics provides a natural probability calculus consistent with context-dependency[2]. Common to those approaches is that the underlying reasons for using a quantum formalism are all "classical," i.e., they do not rely on any actual quantum effects. Thus the common choice of referring to them as "quantum-like."

Among the puzzling aspects of quantum physics, the one that is mainly discussed and accepted in the quantum-like social sciences' literature is contextuality. The reason is that even if nonlocality existed for social sciences, observing it would be impossible with current technology (de Barros and Suppes, 2009). So, even

[1] A classical, i.e., non-quantum, explanation may be found in Moreira and de Barros (2021).

[2] Albeit one that is severely constrained (de Barros, 2012a). We argued elsewhere that if only contextuality is at play, the quantum probability calculus restricts too much the realm of possibilities of social science experiments (de Barros, 2015; de Barros and Oas, 2015).

though nonlocality is sometimes considered, it is always understood differently from the way it is in physics. In physics, nonlocality requires correlations that cannot be explained by causal mechanisms that act in a faster-than-light fashion. In the literature of quantum-like social sciences, nonlocality is simply the violation of probability rules explainable by context dependency. However, this author is unaware of any discussions about the lack of identity and indistinguishability in this literature. This chapter aims to discuss indistinguishability and identity in social sciences and speculate about possible applications of ideas from quantum statistical mechanics in this realm.

This chapter is organized as follows. First, in Section 1.2, I discuss the concepts of identity and indistinguishability, both in classical physics and in quantum mechanics. I then, in Section 1.3, take the strong metaphysical stance that the ontology of quantum particles implies their non-identity, and I examine some of its consequences. In Section 1.4, I examine some proposals that use non-classical statistics outside of physics, suggesting the utility of indistinguishability in the social sciences. In Section 1.5, I argue that economics is another possible field where indistinguishability may be applied, and I speculate about how one could expect a quantum-like lack of identity to affect our descriptions of digital money in economics. Finally, in Section 1.6, I end with some remarks and future perspectives.

1.2 Identity and indistinguishability

This section examines the intimately related concepts of identity and indistinguishability. Let us start with the idea of identity. Intuitively, we have a sense of identity because we identify with ourselves. We then extend this idea of identity to other people. Andrei Krhrennikov has his identity, which is different from mine or this chapter's readers (assuming the reader is not Khrennikov). We can further broaden this idea of identity to animals, such as my pet fish or the squirrel who eats my fruits. For example, my fish is different from other fishes of the same species who live in the river. He may be very similar to all other fishes, but my fish is the one who is in my fish tank. We can also add objects to the universe of entities that have identity. This pen on my desk is different from another pen in my drawer, and they are not the same, despite being almost identical (i.e., the same model). Finally, since we added objects, why stop there? We can confer identity to the atoms that make my pen or the elementary particles that make those atoms. There is nothing wrong with the idea of "things" having an identity, and it even seems intuitive to assume so. In this sense, an object can only be identical to itself, identity being the property of sameness.

The intuitive idea of identity is fine for casual conversations, but what do we mean by having an identity? Let us examine the pen on my desk and the pen in my drawer. If I put each side by side, they look the same. If I am not careful to label them, I can quickly get confused as to which is which. Does that mean they are the same? Probably not. One may have a minor scratch, or may have less ink, or have some manufacturing differences, and so on. There are tiny details that, if carefully examined, would tell us that indeed they are different pens. The idea here is that even though they look the same, the pens are different because they have different properties. Since there will always be a property that two objects do not share, this can be considered a property view of identity. In this view, for an object to be identical to something else, they need to be the same. Them being the same means having all the same properties. So, even if my pens were identical, i.e., if all their properties were the same, they would still be located in different places: one is on my desk and the other in my drawer.

So, it seems that properties offer a reasonable way to define identity: there cannot be two different things having all the same properties. This idea was represented in Leibniz's Principle of the Identity of Indiscernibles (PII). This principle states that x is equal to y if and only if all properties of x are the same as y's. Formally, PII is often expressed as

$$\forall P\,(P(x) \iff P(y)) \iff x = y,$$

where P's are properties of x and y. We emphasize that this representation of Leibniz's PII equates identity with the logical symbol of equality.

Leibniz's law is at the core of classical logic and, consequently, the vast majority of the mathematics used in the natural and social sciences. For example, let us consider the Zermelo–Fraenkel (ZF) axioms that formalize naive set theory (Manin, 2009). In ZF, the only objects are sets, and the only properties of objects are given by the binary predicate "$\in$." Identity shows up as the binary predicate $x = y$, codifying the idea that x and y are the same sets, i.e., the same objects in ZF. This equality comes from first-order logic, which undergirds ZF set theory. For our purposes, in first-order logic, equality satisfies the following axioms:

Reflexivity: $\forall x\,(x = x)$

Substitutivity: $\forall x \forall y\,(\varphi(x) \to \varphi(y))$ for any formula φ where x is a free variable and $\varphi(y)$ as obtained by replacing any occurrences of x with y.

In addition to the logic axioms of equality, ZF adds the axiom of extensionality. The axiom of extensionality states that two sets x and y are the same if and only if they have precisely the same elements. Formally, this can be written as

$$\forall x \forall y\,[\forall z\,(z \in x \iff z \in y) \iff (x = y)]\,.$$

So, identity appears to be related to an ontological aspect of the objects, which are sets in the universe of ZF. First, using the reflexivity axiom from logic, we assume that an object can only be identical to itself. Second, if two sets share all properties, the extensionality axiom implies they are not two objects but one and the same. So, the concept of identity is baked into the foundations of mathematics, with first-order logic and set theory dealing with objects that satisfy Leibniz's PII.

There is one common objection to the ideas expressed above. When talking about objects' properties and equating them with identity, we effectively conflate the epistemic notion of indistinguishability with the ontological notion of identity. Identity, one can argue, is an inherent property of objects: they either have an identity or not. Indistinguishability, on the other hand, seems to be an epistemic concept. It seems to come from our empirical inability to distinguish two things that, in principle, are not the same.

Let us take, for instance, two "identical" twin brothers. They may seem qualitatively identical, in the sense that one may not, at least at first glance, distinguish one from the other, except that one sees a brother to one's right and another to one's left. However, each brother has their own lived experience, peculiarities, and individual (albeit non-perceptible to some people) differences. So, it would be strange to say that they are identical. Perhaps they may differ at the molecular level; perhaps one's synapses reacted differently to slightly different proximal stimuli. Nevertheless, if one were to obtain further information, such as a scar or a memory, one could, *in principle*, distinguish between them. In other words, they seem indistinguishable because one lacks further information about them. They are each an individual because they have their own lived lives, even if everything else was the same.

To summarize our discussions in this section, we saw that identity is an ontological principle. We assign identity and individuality to objects because it seems natural to do so. The pen in my drawer is different from the pen on my desk. They have, in this sense, an identity. Even if I cannot distinguish between them, i.e., even if they are indistinguishable, I can still talk and think about two different pens, one in my drawer and another on my desk. However, attaching identity to objects is a metaphysical move: when we do so, we are stating that there are two pens, and they are different things, even if they are indistinguishable. Furthermore, this ontological principle is reflected in the mathematics used to describe nature. For instance, most modern standard mathematics is formulated within ZF set theory, which uses the concept of identity through Leibniz's principle of identity of indiscernibles. In the next section, we shall see that some results in quantum physics question this ontological assumption that objects are individuals.

1.3 On the ontology of quantum physics

As we mentioned above, one of the most puzzling aspects of quantum theory is that particles seem to lack individuality. This section will examine the issue of indistinguishability of particles and how it seems to imply this lack of individuality of particles.

Let us start with the origins of this discussion in physics. In the early 20th century, physicists quickly realized that the statistics of quantum particles seem to be different from the classical statistics of Boltzman. It all started with a letter that Satyendra Bose wrote to Albert Einstein. This letter included his rejected paper, in which he detailed how, based on the black-body radiation laws discovered by Plank, quantum statistics was different from Boltzman's statistics. Einstein quickly translated the article to German and had it published in *Zeitschrift für Physik*, under Bose's request. The critical idea in Bose's article is that, since two photons of the same wavelength are indistinguishable, then their mathematical description had to accommodate this indistinguishability and be reflected in the statistics.

Let us start with the mathematical description. Assume that we have two particles (say, two photons), and we describe each as $\psi(\mathbf{r}_1)$ and $\phi(\mathbf{r}_2)$, where the r's subscripts refer to a position measurement of particle one or two, and where ψ and ϕ are two distinct wavefunctions. Since the particles are indistinguishable, we cannot know which particle is in the state ϕ and which is in the state ψ. Therefore, the wavefunction for the entire system, composed of particles one and two, needs to be invariant under permutations of the particles. Otherwise, we would distinguish particle one from two. There are two possible forms for this wavefunction, namely

$$\Psi_s = \frac{1}{\sqrt{2}}\left[\psi(\mathbf{r}_1)\phi(\mathbf{r}_2) + \psi(\mathbf{r}_2)\phi(\mathbf{r}_1)\right]$$

or

$$\Psi_a = \frac{1}{\sqrt{2}}\left[\psi(\mathbf{r}_1)\phi(\mathbf{r}_2) - \psi(\mathbf{r}_2)\phi(\mathbf{r}_1)\right].$$

The first form, Ψ_s, is symmetric under permutations of one and two, whereas the second form, Ψ_a is anti-symmetric. The symmetric form corresponds to particles with integer spin, as the case for photons, and are, aptly, called bosons (in honor of Bose). The anti-symmetric wavefunction describes particles with fractional spin, such as electrons, called fermions (after the physicist Enrico Fermi, who formulated the quantum statistical mechanics for electrons).

Interestingly, the above symmetries or anti-symmetries of the wavefunction are based on indistinguishability, an epistemic concept, as we saw, but have real-world consequences. For bosons, this symmetry leads to two bosons occupying the same quantum state. We can have as many bosons as we wish occupying the same state, a phenomenon called Bose-Einstein Condensation. But this phenomenon happens because we cannot count two bosons as two individuals. Instead, we count them as two (indistinguishable) non-individuals.

To illustrate the above point, imagine we have three particles, A, B, and C. We want to distribute them through two possible states, X and Y. Classically, there are eight possibilities: (I) A, B, and C are in X; (II) A and B are in X, but C is in Y; and so on. Since we are talking about statistics, we need to compute the probability of finding a particle or two in X or Y. To do so, we pick one of the particles, say A, and randomly distribute it to either X or Y. Then we repeat the process with B and C. The possible choices are shown in Figure 1.1. From the tree diagram, we can conclude that the probability of having all three particles in state X is $1/8$, two in state X is $3/8$, and so on.

The counting is different if A, B, and C are non-individuals. As shown in Figure 1.2, if we cannot distinguish A, B, and C, we cannot notice a difference between A and B in state X and C in state Y from, say, A being in state Y and B and C in state X. Thus, the probability of all three particles being on state X is not $1/8$ anymore, but the much higher value $1/4$. This probability is $1/4$ because states with two particles in X or two particles in Y are indistinguishable in a fundamental way, as we cannot follow the tree diagram for particles who lack individuality.

Here we find ourselves in an exciting situation. Bose argued that the indistinguishability of quantum particles results in a statistical mechanics different from

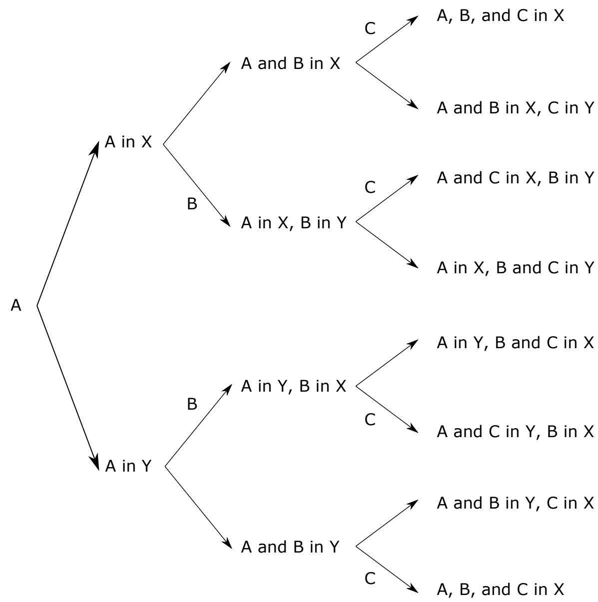

Fig. 1.1 Probability tree diagram for three particles A, B, and C distributed between two states, X or Y.

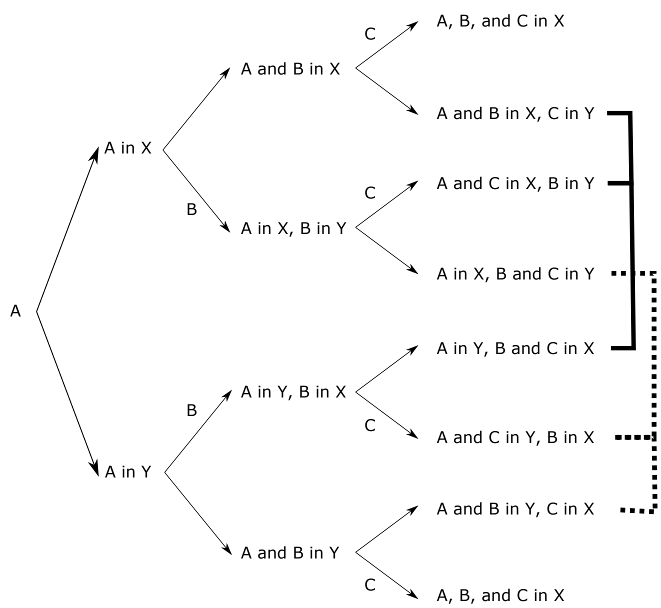

Fig. 1.2 Probability tree diagram for three particles A, B, and C distributed between two states, X or Y. The solid line to the right shows indistinguishable states with two particles in X and one in Y. In contrast, the dashed lines show states with one in X and two in Y. Since they are indistinguishable at a fundamental level, they cannot be counted as different, and they are equiprobable to three particles in either X or Y.

Boltzman's classical one. However, as we saw above, indistinguishability is an epistemic concept: just because we cannot distinguish particle A from B does not mean they are not individuals. However, the counting shown in Figure 1.2 suggests that we cannot treat them as individuals. It is not just that we do not know how to distinguish them, but that we cannot because they lack identity. This suggests that the issue is not epistemic but ontological.

To emphasize this, let us look at the example given by Hermann Weyl. Let us consider a quantum system composed of two "identical" particles (e.g., two electrons) that can be in one of two energy states, E_1 or E_2, $E_1 \neq E_2$. Weyl points out that for such systems, the wavefunction needs to be either symmetric or anti-symmetric. If anti-symmetric, both "individuals" cannot be in the same quantum state, i.e., the only constraint is that the system has total energy $E_1 + E_2$. Because $E_1 + E_2$ is the only possibility, and because the question of which particle has energy E_1 and which has energy E_2 is meaningless, Weyl points out that we count the permutation of 1 and 2 as the same. This symmetry in the permutation of particles should be contrasted with the way of counting possible ways of reaching $E_1 + E_2$ classically. If we have two classical particles, I and M, one can have energy E_1 and the other E_2. Therefore, we may *in principle* figure out which is which, and use this to count two possibilities: (1) I has energy E_1 and M has E_2, or (2) I has energy E_2 and M has E_1. Thus, classically we have two counts, not just one. This difference in statistics, which results in measurable effects (such as Pauli's exclusion principle for anti-symmetric particles), led Weyl to explain this quantum characteristic the following way:

> "That $E_1 + E_2$ occurs only once in [the symmetric case...] means: the possibility that one of the identical twins Mike and Ike is in the quantum state E_1 and the other in the quantum state E_2 does not include two differentiable cases which are permuted on permuting Mike and Ike; it is impossible for either of these individuals to retain his identity so that one of them will always be able to say "I'm Mike" and the other "I'm Ike." Even in principle one cannot demand an alibi of an electron!" Weyl (1950)

Unlike Mike and Ike, quantum particles not only have no identity card, but they seem not to be even distinct individuals. For a quantum system of two identical particles, talking of particle A and B is nothing more than a *façon de parler*, as they only make sense as part of a whole that cannot itself be split or even accounted for as made of two objects with individual properties.

We emphasize that this lack of individuality does not imply that we do not have two "things." For example, one can measure one electron at point A and another at B, and we may be tempted to say that we distinguish them by their location. However, we cannot say that electron "Ike" is in A and "Mike" is in B. All we can say is that one electron is in A and another is in B, but not which one. This impossibility to say which electron is where is reflected in the formal description of the system, as we need to use an anti-symmetric wave function to describe it. So, we have two things, but they lack identity.

Is this non-individuality necessary? Perhaps not. Let us examine what happens to the concept of individuals with one of the most researched realistic interpretations of quantum mechanics: the de Broglie-Bohm pilot-wave interpretation, usually referred to as Bohm's interpretation. Bohm's interpretation uses a classical-like ontology where the individuality of particles is at its core. Though physicists often complain of Bohm's theory as a return to the classical viewpoints of the 19th century, it includes contextuality at its core, through the boundary conditions of the wavefunction solutions. Furthermore, since the Bohmian quantum potential changes instantly, such boundary conditions instantaneously affect the state of a system. Thus, we could hardly say that Bohm's theory is a return to the classical view of the world unless we ignored the development of classical physics in the 19th century.

The effects of indistinguishability of fermions are present as quantum potential effects that keep them apart.[3] In Bohm's theory, as in quantum theory, indistinguishability is imposed ad hoc through the wave function's symmetrization (or antisymmetrization). This assumption about the wave function, introduced by an indistinguishability argument, affects the particles through the quantum potential. We then find ourselves in a weird situation in Bohm's theory: we assume indistinguishability as a *fundamental* aspect of the quantum world affecting the quantum potential, and we then predict real effects, only to deny the reality of indistinguishability through an ontology associated with the physical model. This difficulty is not present with Bohm's theory on contextuality or nonlocality, where the ontology is clearly compatible.

Furthermore, Bohmian mechanics has individuals because, in the theory, one can follow the trajectories of each particle and use those trajectories as an "identity card." These identity cards are obtained within Bohm's interpretation. However, such trajectories cannot be observed. In the lab, subatomic particles do not exhibit such trajectories, and instead show up as discrete clicks in spacetime on the screen of a detector. This observed behavior is different from the classical, as we cannot ensure (unless we believe in Bohm's interpretation) that a particle that we detected here is the same detected there. As Schrödinger puts it,

> "[w]hen a familiar object reenters our ken, it is usually recognized as a continuation of previous appearances, as being the same thing. The relative permanence of individual pieces of matter is the most momentous feature of both everyday life and scientific experience. If a familiar article, say an earthenware jug, disappears from your room, you are quite sure that somebody must have taken it away. If after a time it reappears, you may doubt whether it really is the same one – breakable objects in such circumstances are often not. You may not be able to decide the issue, but you will have no doubt that the doubtful sameness has an indisputable meaning – that there is an unambiguous answer to your query. So firm is our belief in the continuity of the unobserved parts of the string!" Schrödinger (1950, pp. 190)

This is different from what happens with quantum particles, as Weyl pointed out.

We end this section with a comment on our own view on this subject. As we discussed in other papers (de Barros et al., 2017b, 2019; de Barros and Holik, 2020; de Barros et al., 2021), we believe that an ontology of non-individuals is part of the microscopic world. We believe that such an ontology even explains some of the puzzles of quantum mechanics, such as the Bell-EPR apparent paradox if we consider a realistic interpretation. The inconsistency that comes out of a Bell-type inequality has as underlying assumption the individuality of particles (de Barros et al., 2021).

[3] The Bohmian picture is more challenging for bosons (Holland, 1995).

1.4 Indistinguishability outside of physics

In this section, we will speculate whether quantum indistinguishability and its relative, quantum non-identity, have anything to contribute to the modeling of social phenomena. We begin with some discussions on the use of quantum formalisms in social sciences. We then talk about possible applications of quantum indistinguishability.

First, let us distinguish between quantum mechanics and its formalism. Most readers will find this discussion elementary, but, unfortunately, the conflation of those two ideas is a source of much confusion, particularly for those unfamiliar with the literature on quantum social sciences. Quantum mechanics is a fundamental theory in physics that describes microscopic (and some macroscopic) phenomena incredibly accurately. As it was developed, it was meant to describe physical phenomena. Since it is a fundamental theory, it is widely relied on and used outside of physics. For example, one cannot imagine modern chemistry doing without at least some knowledge of quantum mechanics, which is essential to our understanding of the periodic table. To a lesser degree, quantum mechanics also enters the realm of biology, indirectly through chemistry and directly through processes that rely on non-classical dynamics. More directly, the study of quantum processes in biology forms a field of its own, quantum biology. Prominent phenomena studied in quantum biology are magnetoreception, olfaction, photosynthesis, and vision (Marais et al., 2018).

In the early years of quantum mechanics, some prominent physicists hypothesized that quantum processes might be relevant not only to chemistry and biology but perhaps to understand consciousness and the brain. Notably, von Neumann (1955) connected the measurement problem with the mind of an observer (for more details, see Suppes and de Barros (2007)). This connection was later on deepened by London and Bauer (1939) and Wigner (1961). It is not, therefore, surprising that many are still exploring such connections. However, even though some celebrated physicists proposed this idea, most physicists see this connection as unlikely. This attitude is also true for other social quantum phenomena. Though many physicists would find it credible that quantum mechanics may appear in chemical or biological processes, most would view the appearance of quantum phenomena in the social sciences with intense skepticism. The main reason for such skepticism is that social phenomena involve the interaction of very complex systems (humans) in a thermal bath (the environment). In such situations, quantum decoherence would make any quantum effects quickly disappear, and classical descriptions would be possible (Waldner, 2017). However, some prominent researchers, such as Physics Nobel laureates Roger Penrose (1989) and Brian Josephson (2019), still believe there is a connection between quantum mechanics and psychology through consciousness. For a discussion on the different views concerning quantum mechanics and the mind, we direct the reader to de Barros and Montemayor (2019).

Differently from the above view, in the mid-2000s, William Lawless and Peter Bruza discussed the possibility of organizing a conference in social interactions and quantum mechanics. Their idea was that social interactions might be context-dependent in the same way that quantum observables are. In 2007, Lawless and Bruza, together with C. J. van Rijsbergen and Don Sofge, organized the first Quantum

Interactions conference as part of the 2007 AAAI Spring Symposium Series held at Stanford University[4]. This conference was the beginning of several other meetings. It resulted in the creation of a vibrant community whose goal was not only to seek quantum-like phenomena in social interactions but also to investigate if the contextual probability calculus developed by the founders of quantum mechanics could also be useful outside of physics. Its scope covered areas as diverse as computer science, linguistics, econophysics, cognition, and decision-making. According to this view, quantum mechanics should not be seen as a theory of microscopic phenomena applied to the social sciences. Instead, it should be thought of as a contextual calculus whose application transcends the physics of the very small. From the 2007 Stanford conference, a burgeoning field of research emerged, with hundreds of articles and several books published on the subject (see, for example, Busemeyer and Bruza (2012); Haven and Khrennikov (2013, 2017) and references therein). Now, quantum-like models exist in many distinct areas, such as econophysics (Haven, 2002, 2005, 2004), cognition (de Barros, 2012b; Khrennikov and Haven, 2009; Khrennikov et al., 2014; Moreira and Wichert, 2016; Pothos and Busemeyer, 2013), decision-making (Busemeyer et al., 2014, 2009, 2006; Khrennikov, 2009; Haven and Khrennikov, 2016), political sciences (Khrennikova et al., 2012, 2014; Khrennikov, 2016), and linguistics (Bruza et al., 2009, 2015; Aerts et al., 2012), to name a few.

The use of the quantum formalism in social sciences is not far-fetched. First, in this context, the quantum formalism has nothing to do with the quantum mechanics of microscopic systems but instead with particular aspects of social phenomena, especially contextuality (de Barros and Suppes, 2009). In fact, it is possible to create classical models of cognitive processes that exhibit quantum-like characteristics, such as violations of monotonicity in probability theory (de Barros, 2012b; Busemeyer et al., 2017). As such, we can think of the quantum formalism as one of the many different ways to represent complex contextual phenomena[5]. It presents the advantage that it is well developed and studied in the context of quantum physics. Therefore, we use this perspective to think about indistinguishability outside of physics, i.e., that the quantum formalism may offer insights into the social sciences.

Now, attempts to use indistinguishability are not new outside of physics. For example, Bose-Einstein distributions were used in information retrieval (Amati and Van Rijsbergen, 2002). The idea here is that the indistinguishability of “particles,” in this case, linguistic elements, comes from the inability to attach identity to them. This lack of identity comes from the fact that language is contextual, a case that resembles the indistinguishability of quantum properties examined by de Barros et al. (2019). Therefore, cases of linguistic contextuality would be reflected on the deviation of classical statistics toward a Bose-Einstein one. Another use of Bose-Einstein statistics appears in the work of Bianconi and Barabási (2001). Their paper uses the Bose-Einstein statistics to examine complex networks such as the Internet and

[4] See the Preface to de Barros et al. (2017a) for a brief historical account of this conference series.

[5] We have argued elsewhere that perhaps a more general approach, such as extended probability theories, might be better suited than Hilbert-space models (de Barros, 2014, 2015; de Barros et al., 2016). However, this chapter will focus on the quantum route and not examine other extended probabilities.

show that they can form Bose-Einstein condensate states. Finally, another example of using indistinguishability in social sciences is Andrei Khrennikov's "social laser" (Khrennikov, 2016). In his paper, Khrennikov uses an analogy between the inverted population in lasers, where the majority of the atoms are in an excited state, and social energy. He hypothesizes that when the population in a state of high social energy reaches a critical rate, we expect the "stimulated emission" of social energy to surpass the spontaneous emissions, leading to an action amplification by the social agents. Indistinguishability enters into Khrennikov's model the same way that it shows up in Einstein's model of Plank's radiation.

However, where else should we expect indistinguishability to play a role outside of quantum physics. In the next section, we will examine a possible application of indistinguishability in economic theory.

1.5 Indistinguishability in Economics: perspectives

A prime candidate for using indistinguishability is economics. The goal of economics is, among others, to study social interactions that involve value. Classic examples are economic models of supply and demand that describe the relation between prices of goods or services as their availability changes. The medium of exchange is, in most industrialized societies, fiat money. So, let us focus on money.

First, let us start with a brief and oversimplified history of money (the interested reader is referred to some classic texts on the subject, such as Galbraith (2017)). In less complex societies, exchanging of goods or services happens through bartering, i.e., through direct exchanges. For example, someone who wants rum may have a small vegetable farm that produces food. Since their vegetable farm does not produce rum, but their neighbor does, they may agree to a direct exchange of some vegetables for a bottle of rum. Of course, this is a very cumbersome and, sometimes, impractical process. Therefore, some societies developed the concept of a medium exchange for bartering, usually in the form of something widely considered valuable (such as, say, salt or gold). So, instead of our two farmers negotiating every time how many vegetables in exchange for a bottle, they may choose to establish the exchange in terms of, say, grams of salt. This way, even if the vegetable farmer has no vegetables, i.e., in winter, they may still access rum through its exchange with salt. This type of medium of exchange is called *commodity money*, gold coins being one of its best known examples. Commodity money was later replaced with *representative* or *commodity-backed money*, essentially a token for the actual commodity. For example, instead of having a coin made out of gold, one may carry a piece of paper money that has the backing of an authority (usually a government) stating that this particular piece of paper can be, if requested, exchanged by the equivalent amount in gold. This form of money gave way to *fiat money,* as many governments, who usually backed their currency with gold, silver, or platinum, moved from commodity money. In the US, the gold standard was abandoned by Nixon in 1971. As such, fiat money has no value except within a social contract. Finally, we get to *digital money*, the form that most money takes in the US nowadays. Instead of issuing actual bills, as it has done in the past, first attached to a commodity (e.g., gold) and later as fiat money, nowadays central banks issue money electronically in databases.

Commodities, representative, and even fiat money are individuals. Once a bill is printed, it has a serial number. Though two dollar bills have the same intrinsic value, they are not the same, in the sense that they constitute classical individuals. They are exchangeable in the same way that the three classical particles are exchangeable in Figure 1.1, meaning that nothing changes if we replace one with another. Nevertheless, the way we count is that of individuals.

Digital money, on the other hand, is not constituted of individuals. One cannot track each digital dollar, but only how many dollars there are at a specific time in a particular account. In this sense, digital money is countable, as it has cardinality. However, unlike print money, one cannot lay down each bill side by side and map them to the set of integers, as this process requires to keep count of the order in which we are mapping. This lack of ordinality means that it does not make sense to think of them as standard sets: they are not representable by standard mathematics (Krause, 1992). Perhaps the proper way to treat such systems mathematically is to use a theory grounded on quasi-sets.

Quasi-set theory is an extension of set theory to include non-individuals. Set theory can be formalized in first-order logic through Zermelo-Fraenkel's axioms (ZF). Most of modern mathematics is done under ZF set theory, sometimes adding extra axioms, such as the Continuum Hypothesis or the Axiom of Choice (Manin, 2009). In ZF set theory, the only elements are sets (e.g., the empty set $\emptyset$, the set containing the empty set $\{\emptyset\}$, and so on). Quasi-set theory includes ZF plus urelement, i.e., objects that are not sets, m and M. The intuitive idea is that the urelements M correspond to individuals whereas m to objects that are non-individuals. In quasi-set theory, we may have $\{a,a\}$ as different from $\{a\}$ if a is an m-object; the first set has cardinality two whereas the second has cardinality one. However, we cannot talk about ordinality for the quasi-set $\{a,a\}$; how would we distinguish the "first" a from the other?

Another possibility is to use the tricks developed in quantum theory to create a probabilistic theory of indistinguishables. As we saw, in quantum theory, when two particles are indistinguishable, we impose symmetries in their Hilbert space description. A possibility would be to construct Hilbert space models for money and then impose symmetries in their description. As we mentioned above, whether this symmetry is one of bosons or fermions is an empirical question. Nevertheless, since digital money lacks identity, either quasi-sets of a Hilbert space formalism seem reasonable to model it. Another approach is to look at the consequences of non-individuals statistics, as Khrennikov has done for the social laser.

1.6 Final Remarks

In this chapter, we examined the concept of indistinguishability and identity in physics and then argued that money has some of the characteristics of indistinguishable particles. Furthermore, by construction, digital money has cardinality but not ordinality. Therefore, it constitutes a prime candidate for being treated as indistinguishable not only because we cannot know but because it actually lacks identity. This situation is similar to quasi-set theory, where sets with no ordinality but with definite cardinality exist.

Because digital money does not have an identity, they share the same statistics as elementary particles: a statistics of indistinguishability. Whether digital money behaves like bosons or fermions is a question we cannot answer at this point. However, one possible consequence of this indistinguishability of digital money is that models such as Khrennikov's social laser could probably be developed for it. For instance, the fact that massive quantitative easing happened in the US without any increase in inflation suggests a form of coherent "money emission." We believe those are interesting questions for further investigation.

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References

Aerts, D. (2009). Quantum structure in cognition. *Journal of Mathematical Psychology*, 53(5):314–348.

Aerts, D., Broekaert, J., Gabora, L., and Veloz, T. (2012). The Guppy Effect as Interference. In Busemeyer, J. R., Dubois, F., Lambert-Mogiliansky, A., and Melucci, M., editors, *Quantum Interaction*, number 7620 in Lecture Notes in Computer Science, pages 36–47. Springer Berlin Heidelberg.

Amati, G. and Van Rijsbergen, C. J. (2002). Probabilistic models of information retrieval based on measuring the divergence from randomness. *ACM Transactions on Information Systems*, 20(4):357–389.

Bell, J. (1964). On the Einstein-Podolsky-Rosen paradox. *Physics*, 1(3):195–200.

Bell, J. (1966). On the Problem of Hidden Variables in Quantum Mechanics. *Rev. Mod. Phys.*, 38(3):447–452.

Bianconi, G. and Barabási, A.-L. (2001). Bose-Einstein Condensation in Complex Networks. *Physical Review Letters*, 86(24):5632–5635. Publisher: American Physical Society.

Bruza, P., Kitto, K., Nelson, D., and McEvoy, C. (2009). Is there something quantum-like about the human mental lexicon? *Journal of Mathematical Psychology*, 53(5):362–377.

Bruza, P., Kitto, K., Ramm, B. J., and Sitbon, L. (2015). A probabilistic framework for analysing the compositionality of conceptual combinations. *Journal of Mathematical Psychology*, 67:26–38.

Busemeyer, J. and Bruza, P. (2012). *Quantum models of cognition and decision*. Cambridge Univ. Press, Cambridge.

Busemeyer, J., Wang, Z., and Lambert-Mogiliansky, A. (2009). Empirical comparison of Markov and quantum models of decision making. *Journal of Mathematical Psychology*, 53(5):423–433.

Busemeyer, J. R., Fakhari, P., and Kvam, P. (2017). Neural implementation of operations used in quantum cognition. *Progress in Biophysics and Molecular Biology*.

Busemeyer, J. R., Wang, Z., Khrennikov, A., and Basieva, I. (2014). Applying quantum principles to psychology. *Physica Scripta*, 2014(T163):014007.

Busemeyer, J. R., Wang, Z., and Townsend, J. T. (2006). Quantum dynamics of human decision-making. *Journal of Mathematical Psychology*, 50(3):220–241.

de Barros, J. A. (2012a). Joint probabilities and quantum cognition. In Khrennikov, A., Migdall, A. L., Polyakov, S., and Atmanspacher, H., editors, *AIP Conference Proceedings*, volume 1508, pages 98–107, Vaxjo, Sweden. American Institute of Physics.

de Barros, J. A. (2012b). Quantum-like model of behavioral response computation using neural oscillators. *Biosystems*, 110(3):171–182.

de Barros, J. A. (2014). Decision Making for Inconsistent Expert Judgments Using Negative Probabilities. Lecture Notes in Computer Science, pages 257–269. Springer, Berlin/Heidelberg.

de Barros, J. A. (2015). Beyond the Quantum Formalism: Consequences of a Neural-Oscillator Model to Quantum Cognition. In Liljenström, H., editor, *Advances in Cognitive Neurodynamics (IV)*, Advances in Cognitive Neurodynamics, pages 401–404. Springer Netherlands.

de Barros, J. A., Coecke, B., and Pothos, E. (2017a). *Quantum Interaction: 10th International Conference, QI 2016, San Francisco, CA, USA, July 20-22, 2016, Revised Selected Papers*. Springer. Google-Books-ID: Chb_DQAAQBAJ.

de Barros, J. A. and Holik, F. (2020). Indistinguishability and Negative Probabilities. *Entropy*, 22(8):829. Number: 8 Publisher: Multidisciplinary Digital Publishing Institute.

de Barros, J. A., Holik, F., and Krause, D. (2017b). Contextualityy and Indistinguishability. *Entropy*, 19(9):435.

de Barros, J. A., Holik, F., and Krause, D. (2019). Indistinguishability and the origins of contextuality in physics. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*.

de Barros, J. A., Jorge, J. P., and Holik, F. (2021). On the assumptions underlying KS-like contradictions. *arXiv:2103.06830 [quant-ph]*. arXiv: 2103.06830.

de Barros, J. A., Kujala, J. V., and Oas, G. (2016). Negative probabilities and contextuality. *Journal of Mathematical Psychology*, 74:34–45.

de Barros, J. A. and Montemayor, C., editors (2019). *Quanta and Mind: Essays on the Connection between Quantum Mechanics and Consciousness*. Number 414 in Synthese Library. Springer, Cham.

de Barros, J. A. and Oas, G. (2015). Some examples of contextuality in Physics: implications to quantum cognition. In Dzhafarov, E. N., Zhang, R., and Jordan, S. M., editors, *Contextuality, from Quantum Physics to Psychology*, pages 153–184. World Scientific.

de Barros, J. A. and Suppes, P. (2009). Quantum mechanics, interference, and the brain. *Journal of Mathematical Psychology*, 53(5):306–313.

Einstein, A., Podolsky, B., and Rosen, N. (1935). Can Quantum-Mechanical Description of Physical Reality Be Considered Complete? *Physical Review*, 47(10):777–780.

Galbraith, J. K. (2017). *Money*. Princeton University Press. Publication Title: Money.

Haven, E. (2002). A discussion on embedding the Black–Scholes option pricing model in a quantum physics setting. *Physica A: Statistical Mechanics and its Applications*, 304(3–4):507–524.

Haven, E. (2004). The wave-equivalent of the Black–Scholes option price: an interpretation. *Physica A: Statistical Mechanics and its Applications*, 344(1–2):142–145.

Haven, E. (2005). Pilot-wave theory and financial option pricing. *International Journal of Theoretical Physics*, 44(11):1957–1962.

Haven, E. and Khrennikov, A. (2013). *Quantum Social Science*. Cambridge Univ. Press, Cambridge.

Haven, E. and Khrennikov, A. (2016). Quantum probability and the mathematical modelling of decision-making. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 374(2058):20150105. Publisher: Royal Society.

Haven, E. and Khrennikov, A. (2017). *The Palgrave handbook of quantum models in social science*. Springer.

Holland, P. R. (1995). *The Quantum Theory of Motion: An Account of the de Broglie-Bohm Causal Interpretation of Quantum Mechanics*. Cambridge University Press.

Josephson, B. D. (2019). The Physics of Mind and Thought. *Activitas Nervosa Superior*, 61(1):86–90.

Khrennikov, A. (2009). Quantum-like model of cognitive decision making and information processing. *Biosystems*, 95(3):179–187.

Khrennikov, A. (2016). 'Social Laser': action amplification by stimulated emission of social energy. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 374(2058):20150094. Publisher: Royal Society.

Khrennikov, A., Basieva, I., Dzhafarov, E. N., and Busemeyer, J. R. (2014). Quantum Models for Psychological Measurements: An Unsolved Problem. *PLoS ONE*, 9(10):e110909.

Khrennikov, A. Y. and Haven, E. (2009). Quantum mechanics and violations of the sure-thing principle: The use of probability interference and other concepts. *Journal of Mathematical Psychology*, 53(5):378–388.

Khrennikova, P., Haven, E., and Khrennikov, A. (2014). An Application of the Theory of Open Quantum Systems to Model the Dynamics of Party Governance in the US Political System. *International Journal of Theoretical Physics*, 53(4):1346–1360.

Khrennikova, P., Khrennikov, A., and Haven, E. (2012). The quantum-like description of the dynamics of party governance in the US political system. *arXiv:1206.2888*.

Kochen, S. and Specker, E. P. (1967). The Problem of Hidden Variables in Quantum Mechanics. *Journal of Mathematics and Mechanics*, 17:59–87.

Krause, D. (1992). On a quasi-set theory. *Notre Dame Journal of Formal Logic*, 33(3):402–411.

London, F. and Bauer, E. (1939). La théorie de l'observation en mécanique quantique.

Manin, Y. I. (2009). *A Course in Mathematical Logic for Mathematicians*. Springer Science & Business Media. Google-Books-ID: ftK9pUj8SaMC.

Marais, A., Adams, B., Ringsmuth, A. K., Ferretti, M., Gruber, J. M., Hendrikx, R., Schuld, M., Smith, S. L., Sinayskiy, I., Krüger, T. P. J., Petruccione, F., and van Grondelle, R. (2018). The future of quantum biology. *Journal of The Royal Society Interface*, 15(148):20180640. Publisher: Royal Society.

Moreira, C. and de Barros, J. A. (2021). Order Effects in Bayesian Updates. *Proceedings of the Annual Meeting of the Cognitive Science Society*, 43(43).

Moreira, C. and Wichert, A. (2016). Quantum Probabilistic Models Revisited: The Case of Disjunction Effects in Cognition. *Frontiers in Physics*, 4.

Narens, L. (2014). *Probabilistic Lattices: With Applications to Psychology*. World Scientific. Google-Books-ID: Bh23CgAAQBAJ.

Penrose, R. (1989). *The Emperor's New Mind: Concerning Computers, Minds, and the Laws of Physics*. Oxford University Press, Oxford, NY.

Pothos, E. M. and Busemeyer, J. R. (2013). Can quantum probability provide a new direction for cognitive modeling? *Behavioral and Brain Sciences*, 36(03):255–274.

Schrödinger, E. (1950). What is an elementary particle? *Annual Report of the Board of Regents of The Smithsonian Institution*, pages 183–196.

Suppes, P. and de Barros, J. A. (2007). Quantum Mechanics and the Brain. In *Quantum Interaction: Papers from the AAAI Spring Symposium*, Technical Report SS-07-08, pages 75–82, Menlo Park, CA. AAAI Press.

von Neumann, J. (1955). *Mathematical foundations of quantum mechanics; Translated by Robert T. Beyer from the 1932 German*. Princeton University Press, Princeton, NJ.

Waldner, D. (2017). Schrödinger's cat and the dog that didn't bark: Why quantum mechanics is (probably) irrelevant to the social sciences. *Critical Review*, 29(2):199–233. Publisher: Taylor & Francis.

Wang, Z., Solloway, T., Shiffrin, R. M., and Busemeyer, J. R. (2014). Context effects produced by question orders reveal quantum nature of human judgments. *Proceedings of the National Academy of Sciences*, 111(26):9431–9436.

Weyl, H. (1950). *The Theory of Groups and Quantum Mechanics*. Courier Corporation. Google-Books-ID: jQbEcDDqGb8C.

Wigner, E. (1961). Remarks on the mind-body problem. *The scientist speculates*, pages 284–302.