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DIFFRACTION WITH WELL-DEFINED PHOTON TRAJECTORIES: A FOUNDATIONAL ANALYSIS

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We assume: (*i*) Photons are emitted by harmonically oscillating sources. (*ii*) They have definite trajectories. (*iii*) They have a probability of being scattered at a slit. (*iv*) Detectors, like sources, are periodic. (*v*) Photons have positive and negative states which locally interfere, i.e., annihilate each other, when being absorbed. In this framework we are able to derive standard diffraction and interference results. We thereby eliminate in this approach wave-particle duality for photons, and give nonparadoxical answers to standard questions about interference. For example, in the two-slit experiment each photon goes through only one slit.

Key words: photon, interference, trajectories, stochastic-model, QED, quantum-optics.

There have been many papers on optical diffraction, both foundational and technical. Why another one on the foundations? We have several justifications for the present effort. First, it is, we believe, of continuing philosophical interest to give a purely particle theory of diffraction of light. Such particle theories have a long history reaching back to Newton.

Second, the best known particle theories of diffraction, the stochastic mechanics of Nelson and others, and the quantum potential theory of de Broglie-Bohm, are not really suitable for photons. They both depend on dynamical interaction with a field and consequently are focused on the behavior of particles with positive mass. The theory we develop is not dynamical in this sense. Except for local interaction with matter, we assume photons follow linear trajectories.

Third, using our probabilistic theory of photons we can define, in a standard mathematical way, the scalar electromagnetic field. From a philosophical standpoint this reduces the concept of such a field to the probability distribution of photons.

Fourth, although our results agree with those of quantum mechanics for optical diffraction, a point again of philosophical interest is that we avoid the seeming paradoxes of wave-particle duality. For example, does an individual photon somehow go through both slits simultaneously? In our theory, as we spell out later, the answer is negative.

Our main objective is to derive diffraction from the assumption of well-defined photon trajectories. To do this we start with a harmonically oscillating source assuming only knowledge of the mean distribution of emission (Sec. 1). To get a mean probability distribution that is symmetric in the oscillation of the source we introduce positive and negative states of photons. These assumptions are used to derive the mean distribution of photons in free space (Sec. 2) and to define the electric field (Sec. 3). When photons pass through a slit scattering occurs. The resulting diffraction distribution on the screen at a given time is derived (Sec. 4).

To average over time and derive well-known diffraction patterns, we use two additional hypotheses. The first is that positive and negative photons annihilate each other in the process of absorption by a detector. The second is that the process of absorption, like the process of emission, is periodic. We then derive (Sec. 5) a result asymptotically equivalent to the classical definition of intensity. The convergence to an exact equivalence is rapid.

We conclude (Sec. 6) with our response to a number of the conceptual questions often raised about photon diffraction phenomena. The present article continues and considerably extends the foundational analysis begún in Suppes and de Barros (1994).

There are several aspects of the research we are undertaking to be emphasized. In a general way our approach is to analyze the main empirical phenomena characteristic of quantum electrodynamics, but with a stronger ontological commitment than that of QED to a purely particle theory. This viewpoint contrasts sharply with the ontological commitment to fields of Bohm, Hiley and Kaloyerou (1987), Bohm and Hiley (1993) and Kaloyerou (1994). Of course, our ideas are not as yet as thoroughly developed.

Conceptually a key point for us is to introduce from the very beginning two kinds of photons, or, equivalently, two different states for photons, which we label positive or negative. As already remarked, their introduction leads to a natural symmetrization of the expectation density of photons, and also to a local principle of interference. Because only the net excess of positive or negative photons is observable, it is appropriate to call the positive and negative photons *virtual* and thus not necessarily individually observable. Although our concept of virtual photon is not the same as that of QED, we expect common features to be present in our subsequent extension of the present work.

1. SOURCES AND TRAJECTORIES

It is important within the framework of the ideas we are developing to give sources primacy of place over fields. There is already a justification of this in classical electromagnetic theory because, given the sources, the electromagnetic field is uniquely defined, but given the field, the sources cannot be uniquely identified. Of course, our reason for stressing the importance of sources is deeper. We in fact want to *define* the concept of a field and not use it as a primitive concept in our development. In particular, we want to define the concept of field, as will be clearer later on, in terms of the *expectation* density of photons.

In the present paper we take a semi-classical approach to sources. In particular, we assume that we have a harmonically oscillating point source of monochromatic light. It clearly does not represent a detailed theory about how atoms radiate photons.

The simplest assumption for a harmonic oscillating source is to assume that the source can be represented as a function $A \cos \omega t$. However, there are difficulties with this that are easy to pinpoint. What we actually want is a probability distribution of emission of photons from the source. We cannot use a simple cosine function because it assumes negative values. We also want to stress that we do not attempt to characterize at this point a detailed stochastic formulation of the emission of photons by the source, but are content with having only a temporal expectation density at the source. For this purpose we could use the function $A(1 + \cos \omega t)$.

The difficulty with this function is its asymmetric character in relation to the nature of the cosine function. Moreover, there is a more severe conceptual difficulty if we want to define a field from the expectation density. We would have at points where the field is maximum a zero density of photons.

There is an easy and natural way to symmetrize the problem, which we shall adopt, namely, the introduction of positive and negative states, which we discuss from a conceptual standpoint later. In Sec. 3 the difference in the expected number of positive and negative states of photons is used to define the electromagnetic field generated by the source.

So we end up with the following formulation. Let $n_{\pm}(t)dt$ be the expected number of photons in positive or negative states emitted in the interval $(t, t + dt)$, with $t > 0$. Then we assume, in order to symmetrize the expectation density,

$$n_{\pm}(t) = \frac{A}{2}(1 \pm \cos \omega t), \tag{1}$$

with $t > 0$, where A and ω are real constants determined by the oscillating source.

A central aspect of our proposal for a new theory of diffraction is to revive the Newtonian idea of well-defined trajectories for corpuscles, or as we would now say, photons. It was central to Newton's theory that when corpuscles of light are unimpeded they travel in straight lines, as he put the matter at the beginning of *Opticks* (1730, p. 2), "Mathematicians usually consider the Rays of Light to be Lines reaching from the luminous Body to the Body illuminated." In particular, we assume that in free space photons move with velocity c in straight lines. This viewpoint toward trajectories agrees with the heuristic extension of geometrical optics to "diffraction rays" by Keller (1962, 1965).

2. DERIVATION OF EXPECTATION DENSITY IN FREE SPACE

The physical assumptions about photons which we need in this section, partially anticipated above, are the following.

I. Photons are emitted uniformly in all directions from a harmonically oscillating point source.

II. Let $n_{\pm}(t)dt$ be the expected number of photons in positive or negative states emitted in the interval $(t, t + dt)$, with $t > 0$. Then, we assume (as already stated in (1))

$$n_{\pm}(t) = \frac{A}{2}(1 \pm \cos \omega t), \tag{2}$$

with $t > 0$, where A and ω are real constants determined by the oscillating source.

III. In free space, photons move with velocity c in straight lines following classical paths.

IV. In the presence of matter a photon has a positive probability to be scattered or absorbed.

Under these assumptions, the wave-particle duality is eliminated for photons. Individual photons do not have wave properties, but the nonlocal expectation density, which we now define, does.

General probability concepts. We begin with some basic random variables, for which we use spherical coordinates with origin at the source.

$$X(r,\theta,\varphi,t) = \text{number of photons at point}(r,\theta,\varphi)\text{at time } t,$$
$$X_+(r,\theta,\varphi,t) = \text{number of } + -\text{photons at point } r,\theta,\varphi)\text{at time } t,$$
$$X_-(r,\theta,\varphi,t) = \text{number of } - -\text{photons at point}(r,\theta,\varphi)\text{at time } t.$$

Note that

$$X_+(r,\theta,\varphi,t) + X_-(r,\theta,\varphi,t) = X(r,\theta,\varphi,t). \quad (3)$$

At the source. The expected number of photons emitted at t is:

$$E(X_\pm(0,0,0,t)) = n_\pm(t) = A\left(\frac{1}{2} \pm \frac{1}{2}\cos\omega t\right). \quad (4)$$

Expectation density. In Assumption II we use $\frac{1}{2} \pm \frac{1}{2}\cos\omega t$, rather than $\cos\omega t$, to have a density that is nonnegative for all t. Integrating (2) for the interval $(0,t)$, we obtain the expected number $N_\pm(t)$ of photons emitted during the period $(0,t)$.

$$\begin{aligned} N_\pm(t) &= \int_0^t \frac{A}{2}(1 \pm \cos\omega t')dt' \\ &= \frac{A}{2}(t \pm \frac{1}{\omega}\sin\omega t). \end{aligned} \quad (5)$$

We note that the expected number $N(t) = N_+(t) + N_-(t)$ increases with t, because photons are continually created at the source.

We now derive the space-time expectation density $h_\pm(r,\theta,\varphi,t)$, spherically symmetric in the radial distance r from the point source, so we simplify to $h_\pm(r,t)$. For each t, $h_\pm(r,t)$ becomes a probability density if we divide by $N_\pm(t)$. If a photon is emitted at t', $0 \le t' \le t$, then at time t the photon has traveled a distance r, where

$$t - t' = r/c. \quad (6)$$

For a photon ν emitted from the source at time $t' < t$, we have as the conditional probability for its position random variable $Y_t(\nu)$ at time t:

$$P[Y_t(\nu) = (r,\theta,\varphi)|Y_{t'}(\nu) = (0,0,0)] = \frac{1}{4\pi r^2}\delta[(t-t') - \frac{r}{c}]. \quad (7)$$

We know the expectation density at t' of the source:

$$n_{\pm}(t') = \frac{A}{2}(1 \pm \cos\omega t'), \tag{8}$$

so we have for

$$h_{\pm}(r,t) = P[Y_t(\nu) = (r,\theta,\varphi) | Y_{t'}(\nu) = (0,0,0)] n_{\pm}(t') \tag{9}$$

the following:

$$\begin{aligned} h_{\pm}(r,t) &= \int_0^t \frac{A}{8\pi r^2}\delta[t' - (t - \frac{r}{c})](1 \pm \cos\omega t')dt' \\ &= \frac{A}{8\pi r^2}[1 \pm \cos\omega(t - \frac{r}{c})]. \end{aligned} \tag{10}$$

3. DEFINITION OF FIELD

We define the scalar field $\mathcal{E}$, generated by the spherically symmetric point source in terms of the expectations of the random variables X_+ and X_-, or, put another way, in terms of the photon expectation densities h_+ and h_-,

$$\mathcal{E} = \frac{\mathcal{E}_0(h_+ - h_-)}{\sqrt{h_+ + h_-}}, \tag{11}$$

where $\mathcal{E}_0$ is a scalar physical constant. Using (10),

$$h_+ - h_- = \frac{A}{4\pi r^2}\cos\omega(t - \frac{r}{c})$$

and

$$h_+ + h_- = \frac{A}{4\pi r^2};$$

thus, from (11),

$$\mathcal{E} = \mathcal{E}_0\sqrt{\frac{A}{4\pi r^2}}\cos\omega(t - \frac{r}{c}). \tag{12}$$

Applying the standard definition of average intensity, we get the expected result:

$$I = \langle\mathcal{E}^2\rangle = \frac{\mathcal{E}_0^2 A}{8\pi r^2}. \tag{13}$$

There is a conceptual point to be emphasized in connection with equation (11). This equation is a definition. It shows how we define the concept of an electromagnetic field solely in terms of photons and their expectation density. Hence in the present context the field has no independent physical reality.

On the other hand, the field $\mathcal{E}$ defined by (11) satisfies the three-dimensional wave equation

$$\nabla^2 \mathcal{E} - \frac{1}{c^2}\frac{\partial^2 \mathcal{E}}{\partial t^2} = 0, \tag{14}$$

which for the spherically symmetric case has the following form in spherical coordinates

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \frac{\partial \mathcal{E}}{\partial r}) - \frac{1}{c^2}\frac{\partial^2 \mathcal{E}}{\partial t^2} = 0, \tag{15}$$

where the terms involving θ and φ are eliminated by symmetry.

It is easy to check that the definition of the field given by (11) satisfies the wave equation when $h_{\pm}$ is generalized to an arbitrary number of spherically symmetric sources.

4. DERIVATION OF DIFFRACTION THROUGH A SLIT

The standard two-dimensional geometry of the single slit, to which we restrict ourselves in this and the next section, is shown in Fig. 1, where for the illustrated path x is the coordinate on the axis of the slit and y is the coordinate on the screen. We assume as a local phenomenological model that the probability $P_x(s)$ of a scattering event at any point x of the slit is a decaying exponential function as we move toward the center of the slit away from the edges,

$$P_x(s) = e^{-\lambda(x+a)}(\eta(x+a) - \eta(x)) + e^{+\lambda(x-a)}(\eta(x) - \eta(x-a)), \tag{16}$$

where $a, \lambda > 0$ and

$$\eta(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0. \end{cases}$$

Also $P_x(\bar{s}) = 1 - P_x(s)$, where $\bar{s}$ is the event of no scattering at x. The parameter λ will be a function of the barrier and can be estimated from experimental data.

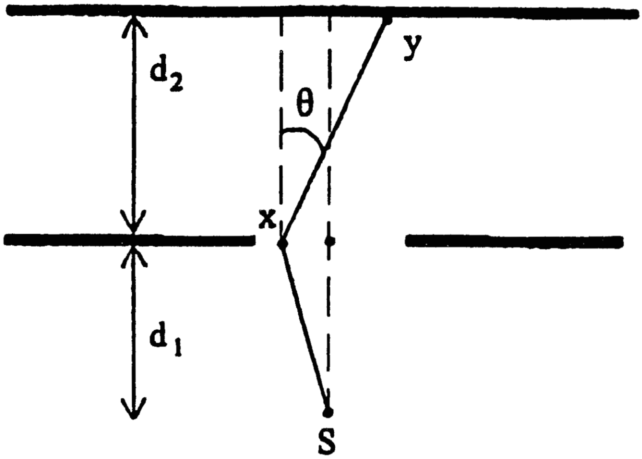

Fig. 1. Typical path in single-slit diffraction.

We define $p_x(y|s)$ as the conditional probability that a photon scattered at x arrives at y on the screen and $p_x(y|\bar{s})$ as the conditional probability that a photon *not* scattered at x arrives at y. We note first that

$$p_x(y|\bar{s}) = \delta(y - \frac{x(d_1 + d_2)}{d_1}), \tag{17}$$

which is just the geometrical shadow.

Second, we assume a cosine law for forward scattering

$$p_x(y|s) = c_2 \cos\theta \tag{18}$$

and we neglect any backward scattering. From Fig. 1 it is clear that

$$\theta = \arctan\frac{|y - x|}{d_2} + \arctan\frac{|x|}{d_1}. \tag{19}$$

Returning again to Fig. 1, let $\Delta t = t - t'$ be the time for the photon to go from the source S to y. We then have

$$t' = t - \frac{1}{c}\left(\sqrt{x^2 + d_1^2} + \sqrt{(y - x)^2 + d_2^2}\right) = g(t, x, y). \tag{20}$$

Using (9) and (20) and integrating across the slit the various conditional probabilities already defined, we get as a general expression

$$h_\pm(y, t) = \frac{1}{2a}\int_{-a}^{a} [p_x(y|s)P_x(s) + p_x(y|\bar{s})P_x(\bar{s})]\, n_\pm(g(t, x, y))dx. \tag{21}$$

Using (8) and (16)–(21), we obtain as the explicit expression for diffraction through a single slit

$$
\begin{aligned}
h_{\pm}(y,t) = {} & \frac{c_2 A}{2a}\int_{-a}^{a}\Big[(e^{-\lambda(x+a)}(\eta(x+a)-\eta(x)) \\
& + e^{+\lambda(x-a)}(\eta(x)-\eta(x-a)))\Big] \\
& \cdot \cos(\arctan\frac{y}{d_2}+\arctan\frac{|x|}{d_1})\left\{1 \pm \cos\omega\left[t-\frac{1}{c}\left(\sqrt{x^2+d_1^2}\right.\right.\right. \\
& \left.\left.\left.+\sqrt{(y-x)^2+d_2^2}\right)\right]\right\}dx \\
& + \frac{A}{2a}\int_{-a}^{a}(1-e^{+\lambda(x+a)}(\eta(x+a)-\eta(x)) \\
& - e^{-\lambda(x-a)}(\eta(x)-\eta(x-a))) \\
& \cdot \delta(y-\frac{x(d_1+d_2)}{d_1})\left\{1\right. \\
& \left.\pm \cos\omega\left[t-\frac{1}{c}\left(\sqrt{x^2+d_1^2}+\sqrt{(y-x)^2+d_2^2}\right)\right]\right\}dx.
\end{aligned}
\tag{22}
$$

We now introduce several approximations for computational purposes: (i) since x is small, we approximate θ by $\arctan(|y|/d_2)$; (ii) we assume that $\cos\theta$ varies slowly compared to $\cos(\omega g(t,x,y))$ since $w \gg 1$; (iii) $P_x(s) = 1$ for all points x of the slit. Using these approximations, we obtain from (22)

$$
\begin{aligned}
h_{\pm}(y,t) = {} & \frac{c_2 A}{4a}\cos(\arctan\frac{y}{d_2}) \\
& \int_{-a}^{a}\left\{1 \pm \cos\omega\left[t-\frac{1}{c}\left(\sqrt{x^2+d_1^2}+\sqrt{(y-x)^2+d_2^2}\right)\right]\right\}dx.
\end{aligned}
\tag{23}
$$

The corresponding expression for two slits with width d of the central barrier is easily obtained from (23):

$$h_{\pm}(y,t) = \frac{c_2 A}{4a} \cos\left(\arctan\frac{y}{d_2}\right)$$
$$\left\{\int_{\frac{d}{2}}^{\frac{d}{2}+a}\left\{1 \pm \cos\omega\left[t - \frac{1}{c}\left(\sqrt{x^2+d_1^2} + \sqrt{(y-x)^2+d_2^2}\right)\right]\right\}dx\right.$$
$$\left.+\int_{-\frac{d}{2}}^{-\frac{d}{2}-a}\left\{1 \pm \cos\omega\left[t - \frac{1}{c}\left(\sqrt{x^2+d_1^2} + \sqrt{(y-x)^2+d_2^2}\right)\right]\right\}dx\right\}. \tag{24}$$

By using a second-order Taylor expansion of x around 0, i.e., by using the linear and quadratic terms of the expansion, we obtain from Maple's symbolic computation program an approximate expression for (23) in terms of the standard Fresnel integrals:

$$h_{\pm}(y,t) = \frac{c_2 A}{4a} \cos(\arctan\frac{y}{d_2})$$
$$\cdot\{2a \pm \sqrt{\frac{\pi}{2\omega\gamma}}[(\sin k)S(\sqrt{\frac{\omega}{2\pi\gamma}}(\beta - 2\alpha\gamma)) - (\cos k)C(\sqrt{\frac{\omega}{2\pi\gamma}}(\beta - 2a\gamma))$$
$$+ (\sin k)\; S(\sqrt{\frac{\omega}{2\pi\gamma}}(\beta + 2a\gamma)) + (\cos k)\; C(\sqrt{\frac{\omega}{2\pi\gamma}}(\beta + 2a\gamma))]\}, \tag{25}$$

where

$$\alpha = \omega\left(t - \frac{d_1 + \sqrt{y^2+d_2^2}}{c}\right), \tag{26}$$

$$\beta = \frac{\omega y}{c\sqrt{y^2+{d_2}^2}}$$

$$\gamma = \frac{\omega}{c}\left(\frac{1}{2d_1} + \frac{d_2^2}{2(y^2+d_2^2)^{\frac{3}{2}}}\right), \tag{28}$$

$$k = \omega\left(\frac{\beta^2}{4\gamma} + \alpha\right), \tag{29}$$

$$C(x) = \int_0^x \cos(\frac{\pi}{2}z^2)dz, \tag{30}$$

$$S(x) = \int_0^x \sin(\frac{\pi}{2}z^2)dz. \tag{31}$$

A similar approximation in terms of Fresnel integrals for equation (24) is also directly computable by Maple, but the expression is complicated, so we omit it. The integral of equation (23) is similar to a classical diffraction integral derived in Sommerfeld (1964). By using his approximations we can derive his result.

5. ABSORPTION OF PHOTONS

There are two important conceptual assumptions we now introduce. They are essential to our analysis of absorption, but were not required for the derivation of (22), i.e., of the expression at the screen of the expectation density $h_{\pm}(y,t)$. These new assumptions are required for a proper formulation of the second-order effects that are characteristic of absorption, as classically measured by the square of the field, averaged over time.

First, we assume that the absorber, or photodetector, itself behaves periodically with a frequency ω. Thus, the probability p_A of absorbing a photon is, independent of y:

$$p_A = c_3(1 + \cos(\omega t + \psi)), \tag{32}$$

where ψ is an arbitrary phase that can be randomized.

Second, we assume a *local* principle of interference. As already remarked, during the process of absorption positive and negative photons cancel each other in pairs, so the effective absorption in a short interval is the net surplus, if any, of either positive or negative photons.

We simplify the analysis, without loss of conceptual content, by analyzing two point sources, which we can think of as an approximation to two slits. So we have from the two coherent point sources at y and t

$$h_{\pm}^{(1)}(y,t) = B(y)(1 \pm \cos(\omega t + \varphi_1(y))), \tag{33}$$

$$h_{\pm}^{(2)}(y,t) = B(y)(1 \pm \cos(\omega t + \varphi_2(y))), \tag{34}$$

and

$$h_{\pm}(y,t) = h_{\pm}^{(1)}(y,t) + h_{\pm}^{(2)}(y,t). \tag{35}$$

Because only the relative phase is observable, without loss of generality we may set $\varphi_1(y) = 0$ and $\varphi_2(y) = \varphi(y)$.

We show that for two point sources differing by an arbitrary phase φ, the time-averaged imbalance of expected number of positive and negative photons absorbed converges rapidly to the classical intensity of the defined field. First, the expected number $E(\#\pm)$ of each type of photon absorbed is the time-averaged product:

$$E(\#\pm) = \langle h_{\pm}(t)p_A(t)\rangle. \tag{36}$$

The averaging is required because an absorption of an individual photon by an atom of a photodetector takes on average several orders of magnitude longer than the mean optical period of the photons, both theoretically and experimentally (Nussenzveig, 1973).

From (32)-(36), we obtain

$$E(\#\pm) = \frac{Bc_3}{T}\int_0^T (2 \pm \cos\omega t \pm \cos(\omega t + \varphi))(1 + \cos(\omega t + \psi))dt. \quad (37)$$

We now observe for $T \gg 1/\omega$, where $\omega \approx 10^{15} s^{-1}$,

$$\frac{Bc_3}{T}\int_0^T \cos\omega t dt \approx 0 \quad (38)$$

and

$$\frac{1}{T}\int_0^T \cos^2 \omega t dt \approx \frac{1}{T}\int_0^T \cos^2(\omega t + \varphi)dt. \quad (39)$$

Using (37), (38) and (39), we obtain

$$\begin{aligned}&|E(\#+) - E(\#-)| \\ &\approx \frac{Bc_3}{T}\left|\int_0^T 2(\cos\omega t \cos(\omega t + \psi) + \cos(\omega t + \varphi)\cos(\omega t + \psi))dt\right| \\ &\approx Bc_3|\cos\psi + \cos(\varphi - \psi)|.\end{aligned} \quad (40)$$

Randomizing between the two corresponding phases of $\psi = \varphi$ and $\psi = 0$ in the absorber, we see from (40) that the outcome is the same, namely,

$$|E(\#+) - E(\#-)| \approx Bc_3(1 + \cos\varphi). \quad (41)$$

Defining the field at the fixed point y, using (11), we have

$$\mathcal{E} = \mathcal{E}_0\sqrt{B}(\cos\omega t + \cos(\omega t + \varphi)). \quad (42)$$

Squaring $\mathcal{E}$ and averaging over time, we obtain the intensity I at a point y:

$$I \approx \mathcal{E}_0^2 B(1 + \cos\varphi). \quad (43)$$

Comparing (41) and (43) we see that for fixed phase difference φ of the two sources we get that the intensity of the field is proportional to the expected total number of photons, i.e., $E(\#+) + E(\#-)$.

We now consider two special cases, one of positive and one of negative interference. If $\varphi = 0$ in (41), so that the two sources have the same phase, then

$$|E(\#+) - E(\#-)| = 2Bc_3. \tag{44}$$

If $\varphi = \pi$, the case of negative interference, then

$$|E(\#+) - E(\#-)| = 0. \tag{45}$$

So in this case we get a classical case of complete destructive interference where no photons are detected, but by using only our purely local principle of interference.

We are now in a position to explain one of the most seemingly paradoxical aspects of the two-slit experiment. How can it be, so the paradox goes, that by closing one slit (or turning off one source), the number of detected photons increases (Feynman, 1985)? Our answer is simple, as can be seen from (45). When one slit is closed we eliminate the destructive local interference. So, with only one slit open, the expected number of photons detected is Bc_3, whereas, as we have shown, with both slits open, the expected number is 0.

6. SOME STANDARD QUESTIONS AND ARGUMENTS

It is instructive to see how the concept of photons with definite trajectories leads to nonparadoxical answers to the standard questions raised about diffraction and interference, especially in the context of quantum mechanics.

1. Where are the waves? In the theory of photons proposed here an individual photon does not have wave properties. The expectation density does, for it inherits the wave properties of the oscillating source.

2. Interference with one photon at a time? Yes, but. Given sufficient time T a very low intensity beam will produce the same pattern of diffraction or interference as a very strong beam for a short time T'. The required computation of the times T and T' is straightforward in principle: The same expected number of photons should be emitted from sources with differing intensities for the two periods of time. But for extremely low intensity beams the expected density is a poor approximation to the real phenomena.

In principle, we can explain the paradox of a single photon interfering with itself. The "single photon" of the paradox is the observable event at a detector. The destructive interference that leads to no observed event is explained by a positive and negative photon, in our sense, annihilating each other. In other words, the net excess of positive or negative photons is zero. As is apparent from the explanation, the total number of photons, positive or negative, does not correspond to the total number of observable events, which is another reason to call our positive and negative photons virtual. Other aspects of photon interference are discussed in Suppes and de Barros (1994).

3. How can the intensity at a point on the screen be increased by closing one of the two slits? See our answer after Eq. (45).

4. Without waves, why do particles hit the screen outside the geometrical shadow? In terms of the basic assumptions of this paper, the answer is clear. The scattering of photons by the matter of the barrier can change their trajectories to lie outside of the shadow.

5. Does an individual photon ever go through both slits simultaneously? No. Since our basic assumption is that photons have a well-defined trajectory, our negative answer is obvious. Of course, our analogue of a wave concept, the expectation density, can, and generally will, be simultaneously nonzero at both slits.

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REFERENCES

D. Bohm and B. J. Hiley, *Undivided Universe: Ontological Interpretation of Quantum Theory* (Routledge, New York, 1993).

D. Bohm, B. J. Hiley, and P. N. Kaloyerou, *Phys. Rep.* **144**, 349 (1987).

R. P. Feynman, *QED: The Strange Theory of Light and Matter* (Princeton University Press, Princeton, 1985).

P. N. Kaloyerou, *Phys. Rep.* **244**, 287 (1994).

J. B. Keller, *J. Opt. Soc. Am.* **52**, 116 (1962).

J. B. Keller and D. B. Hansen, *Acta Physica Polonica* **27**, 217 (1965).

I. Newton, *Opticks, 4th Edition* (William Innys, London, 1730).

H. M. Nussenzveig, *Introduction to Quantum Optics* (Gordon & Breach, New York, 1973).

A. Sommerfeld, *Optics. Lectures on Theoretical Physics, Vol. IV* (Academic, New York, 1964), pp. 199–204.

P. Suppes and J. Acacio de Barros, *Int. J. of Theor. Phys.* **33**, 179 (1994).