On Quantum Mechanics and the Mind

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1 Introduction

This paper is an extension of the main points I started to discuss during the Foundations of Mind Conference, organized by Sean O'Nuallain and held in Berkeley in March 2014. I was fortunate to be invited by Sean to chair a section where Henry Stapp would talk about his theory of quantum mechanics and the mind. But Sean also gave me the almost impossible task of criticizing Stapp's views. Of course, as soon as I realized that one can criticize without actually giving a plausible alternative, the task became less daunting. So, the goal of this paper is to put forth a couple of criticisms of the quantum mind theory and, perhaps, suggest a possible alternative of what theory could replace at least certain aspects of it.

Before going on, let me start with a general comment. I do not wish to claim here that Stapp's theory of quantum mind is without merits. I find it a fascinating way to think about the mind and its connection to quantum mechanics, a subject that has fascinated me ever since I was a graduate student and got in touch with many texts on the subject, including von Neunann's seminal book [32]. My goal here is to point out some difficulties that I see with the theory, both from a conceptual as well as a technical point of view, and to question it as a way to help with the problem of consciousness. This will be done, hopefully, without resorting to too many technical arguments, and some liberty will be taken about some details. I will, however, assume that the reader is familiar with quantum mechanics and its formalism and notation at the level of standard textbooks, such as [4].

I organize this paper in the following way. The first part of my argument is presented in section 2, I discuss the von Neumann approach to quantum mechanics, the backbone of Stapp's theory. In it, I try to argue that von Neumann's views, albeit *consistent* with empirical evidence, loose their motivation when viewed from a modern perspective. Motivation, of course, is in the eyes of the beholder, and in this section I cannot hope but only convince those who were already skeptical of the quantum-mind theory. However, revisiting von Neumann's theory is useful for my more focused criticisms, spelled out in the next section. In section 3 I quickly introduce some main aspects of Stapp's theory (detailed in [22] on this proceedings), as I (hopefully) understand it, and I present two arguments against it. One argument is more specific to the particular application of QZE shown in [22]. The other argument is broader, and questions the proposed use of QZE to solve the mind/matter problem. In particular, we show that Stapp's use of QZE leads to a circularity in his argument, and therefore is not really a solution to the mind/matter problem. Because this presents serious difficulties to the proposed model, it makes it the most relevant point in my paper. Finally, in section 4 I end the paper on a more positive note, giving some possible alternative approaches that are somewhat in the spirit of what Stapp is trying to achieve with his theory of the quantum mind.

2 von Neumann's Approach

Let me start with the overall "historic" argument about the connection between the mind and quantum mechanics, first put forth by von Neumann himself in his discussions of the theory of measurement [32]. According to the Copenhagen interpretation, one of the difficulties of quantum mechanics, first pointed out clearly by Bohr [1], is the dual nature of the evolution of a physical system. On the one hand quantum mechanics is deterministic: given the state $|\psi_0\rangle$ of a system at time t_0, its time evolution is given by

$$|\psi\rangle = e^{-\frac{i}{\hbar}\hat{H}t}|\psi_0\rangle, \tag{1}$$

where $\hat{H}$ is the Hamiltonian operator. Equation (1) uniquely determines the system $|\psi\rangle$ at time $t \geq t_0$. On the other hand, quantum mechanics, through the measurement process, is probabilistic: all we can talk about from the $|\psi\rangle$ are the probabilities $P(o_i) = |\langle o_i|\psi\rangle|^2$ of possible outcomes o_i of an experiment $\mathcal{O}$ represented by the observable $\hat{O}$, such that $\hat{O}|o_i\rangle = o_i|o_i\rangle$. So, it seems that there are two different types of incompatible evolution in quantum mechanics, and one of them is associated to a very special type of interaction: a measurement.

The question posed by the founders of quantum mechanics was the following: what makes measurements different? To Bohr, a measurement was an interaction with a classical system. This is all good when we are talking about, say, an electron (clearly quantum) and dark spots on a photographic paper (clearly classical). But the problem becomes trickier when the measurement device itself is small: for instance, when we think of atoms as photodetectors, or when we deal with mesoscopic systems. Some measurement devices are clearly classical, whereas others are not. So, where is this boundary between classical and quantum physics? This border is what was known as the Heisenberg cut, the point where anything over it behaves classically.

Von Neumann considered this idea of a dual dynamics, one for classical measurement apparatus and another for quantum systems, unsatisfactory [32]. To overcome this, he treated both the observed system *and* the measurement apparatus as quantum systems. A measurement consisted thus of something with the following characteristic. Let $\hat{O}$ be the observable represented by the (quantum-mechanically described) measuring apparatus $\mathcal{O}$, such that if a system is initially in the state $|o_i\rangle$ and the apparatus in the state $|0\rangle$ (i.e., the state is represented by the value of the pointer on the measuring device measuring nothing, the reset position), then the interaction leads the the following evolution:

$$|o_i\rangle \otimes |0\rangle \rightarrow |o_i\rangle \otimes |i\rangle,$$

where $|i\rangle$ means the apparatus is pointing to the value corresponding to the outcome o_i. This is all fine with states for the system that are eigenstates of the measurement apparatus. However, if the system is in a superposition of the type

$$|\psi\rangle = c_1|o_1\rangle + c_2|o_2\rangle,$$

$|c_1|^2 + |c_2|^2 = 1$, the interaction with the measurement apparatus leads to

$$|\psi\rangle \otimes |0\rangle \rightarrow c_1|o_1\rangle \otimes |1\rangle + c_2|o_2\rangle \otimes |2\rangle.$$

Notice that this last equation is not what happens with a measurement process, where we either get one outcome or the other. Instead, because of the linearity of quantum evolution, superpositions of the initial state lead to, after an interaction with the measurement device, a quantum superposition of the whole system. In other words, there is no collapse of the wave function, and therefore, no "measurement", in the sense of Bohr, happened. From the linearity of quantum mechanics, von Neumann observed that the quantum superposition of system and apparatus could be extended all the way to larger and larger systems. However, there was one point where unambiguously there was no superposition: the mind of an observer. This is the case because we *never* see the superposition of, say, Schrödinger's cat dead and alive. So, von Neumann posited that the actual irreversible aspect of the quantum measurement, the collapse of the wave function, happens at the mind, thus avoiding the issue of determining Heisenberg's cut.

Thus, von Neumann's idea that the Heisenberg cut, i.e. where does quantum interference is lost and classical outcomes exist, is done in the interaction between the mind and the systems correlated to the original quantum system being measured. For example, an atom is detected by a detector, which is then observed by the researcher's eyes, which send signals to the brain. At each step, atom, detector, and so on, we have a chain of entangled quantum systems. It is not until this chain interacts with the mind (somewhere after the eyes) that a collapse of the wave function actually happens.

The main motivation for von Neumann to go "all the way up" to the mind was the lack of clear boundary between the classical and the quantum regimes. However, as we know from modern environmental decoherence theory[1], such

[1] Some might point that von Neumann already accounted for decoherence in his book, as he talks about the possibility for off-diagonal elements of the partial trace of the density matrix to go to zero because of a measurement process. We clarify that what we are talking about here is the dynamical theory of decoherence that, for instance, makes explicit claims about how the off-diagonal terms of the density matrix go to zero, a result that has practical implications for the construction of quantum computers. This theory's predictions have been observed, for example, in mesoscopic systems [2].

boundary is not as muddled as initially thought [19]. Furthermore, quantum superposition, the cornerstone of quantum effects, is not the norm for larger systems, and interference effects associated to such superpositions decay extremely rapidly even for mesoscopic systems, in accordance with the theory [2].

Now, what does environmental decoherence has to tell us about von Neumann's quantum mind? First of all, it does not solve the measurement problem, the main reason that led von Neumann to his interpretation, even if we accept Bell's concept of solving For All Practical Purposes (FAPP). Because environmental decoherence still relies on quantum evolution, it still carries all the way up the same superpositions that have troubled physicists for a century.

Second of all, though decoherence clarifies the Heisenberg cut, it is by no means a disproof of von Neumann's theory. Because the underlying dynamics is still linear, one can argue that quantum superpositions still exist, and that the "collapse" only happens in the mind[2]. If not anything else, decoherence probably makes it impossible to falsify von Neumann's ideas, as there is no way quantum coherence can be held up to the, say, brain level [30], in an observable way.

But decoherence does tell us that we can *talk* about the Moon, for all practical purposes, even if we have not observed it, and explains why most macroscopic things behave in a seemingly classical way. So, in a certain sense, it makes von Neumann's theory more far-fetched. Notice that von Neumann proposed a solution to the measurement problem by denying a dual dynamics of quantum and classical evolution for different systems, but instead by creating a new system, not subject to the laws of quantum mechanics, that had its own dynamics as well: the mind. So, it simply replaces a mystery by another mystery, without adding any explanatory power.

More importantly, decoherence removes the main motivation for von Neumann's collapse postulate. If we recall, this postulate was introduced (by Heisenberg) to deal with the fact that we do not see macroscopic quantum superpositions. But let us say that we do indeed have, as von Neumann says, a macroscopic quantum superposition getting all the way to the mind. Everett's many worlds/minds interpretation simply says that our mind makes a selection, among the many possible, but we still have a superposition [11]. What selection does the mind make? Not any of the infinite amount of possible experimental outcomes, but instead such selections that are consistent with the preferred pointer basis of the involved quantum observables, as given by the decoherence of the quantum states. So, no need for any special dynamics of the mind outside of the quantum (or physical) realm.

3 Stapp's Quantum Mind

I now turn to Stapp's approach [21], more specifically his arguments laid out in [22]. Let us assume that von Neumann's theory is correct, and that there is a

[2] This is possibly the main reason why so many of the proponents of decoherence as a way to clarify the measurement problem are sympathetic to the many worlds/minds interpretation of quantum mechanics.

different entity, the mind, that does not satisfy the laws of quantum mechanics, and is responsible for the classical character of the measurement. Stapp poses the interesting question of whether such entity could affect the physical world, a huge problem for the proponents of the mind/brain duality.

Stapp suggests the Quantum Zeno Effect (QZE) [17] as a possible mechanism for the mind to affect matter. In the original QZE, it was shown that if we continuously observed an unstable particle, this particle would not decay. However, we can modify this argument, and show that by continuous observations (or many observations close to each other) we can make a particle change a quantum state. To use Stapp's example in [22], let us start with a coherent state with amplitude α, represented by the ket $|\alpha\rangle$. Now, let us perform a simple yes/no experiment, where the question being asked is whether the system is a coherent state with amplitude $\alpha + \Delta$, where Δ is small compared to α. Because coherent systems are not orthonormal, the probability for the state $|\alpha\rangle$ to be in $|\alpha + \Delta\rangle$ is nonzero, and given by

$$P\left(|\alpha + \Delta\rangle \| \alpha\rangle\right) = |\langle\alpha|\alpha + \Delta\rangle|^2 \approx 1 - \Delta^2 \tag{2}$$

for Δ^2 sufficiently small. Furthermore, if we make N several successive measures, each time asking the question with a larger amplitude (by a factor Δ), the probability of observing $\alpha + N\Delta$ is given by

$$P\left(\alpha + N\Delta\right) \approx \left(1 - \Delta^2\right)^N \approx 1 - N\Delta^2,$$

and *not* $|\langle\alpha|\alpha + N\Delta\rangle|^2$. Thus, making Δ very small, i.e. by almost continuously observing a quantum harmonic oscillator in the semi-classical coherent state, it is possible to increase its amplitude of oscillation.

In [22], Stapp applies this ideas to motor cortex measurements performed by Rubino, Robbins, and Hatsopoulos [20]. His idea is that in the same way that the mind causes the collapse of the wave function, the effect of the mind "observing" a system can make it change its state from $|\alpha\rangle$ to $|\alpha + N\Delta\rangle$. I see two problems with this model, one conceptual and one specific to the application.

The specific problem is more technical in nature, and perhaps not as important as the second, so let me talk about it first. Stapp starts with the magnetic field in what he claims is a computational unit: the minicollumn. Because the magnetic field is very weak, of the order of pT, it follows that if we model its oscillations with a quantum harmonic oscillator in a coherent state (as the computations above), it is justifiable to use quantum mechanics, as α is only at the order of 10^1. However, why talk about the magnetic field? Given the low frequencies, the magnetic and electric fields are uncoupled, and the regime is essentially a quasi-static one [18]. Furthermore, as [20] show, the electric field involved in the process is at the order of 10 μV, and its energy is about 10 orders of magnitude that of the magnetic field. More importantly, because it is a much stronger field, it not only is more relevant to understand the processes, but it is also describable, to a very good approximation, by classical equations, as one should expect. So, to make his model stick, Stapp would have to clearly justify the picking of magnetic fields over electric fields, and more importantly, how such fields, when increased even tenfold, would affect the dynamics of the brain in a significant way (given that it carries energy that is about 10^9 times less than the electric field).

We now turn to the main problem with Stapp's approach. As we mentioned above, Stapp starts with a weak magnetic field, modeled by a coherent state, which is subjected to successive measurements. The mind is seen affect the field itself through the QZE, a (perhaps measurable) mechanism for the mind/brain interaction. In other words, if the mind chooses to measure $|\alpha + \Delta\rangle$, and then $|\alpha + 2\Delta\rangle$, then $|\alpha + 3\Delta\rangle$, and so on, then it can make the amplitude of the field increase, within a reasonable amount of time, from α to $\alpha + N\Delta$.

But what is a measurement of $|\alpha + \Delta\rangle$ or $|\alpha + 2\Delta\rangle$? In von Neumann's formulation, this process is done by the presence of a physical system, i.e., by some hardware (responsible, as shown by decoherence, for the choice of a pointer basis). For example, to measure the spin of an electron, we have to produce a Stern-Gerlach experimental setup before the atoms get to the photographic place, which then sends photons to the eyes of the observer, that activate neurons in the brain, and somewhere or somehow finally gets to the mind. But the Stern-Gerlach setup needs to be there; it cannot be produced by the mind. The same is true for the QZE. A *physical* measurement has to be made to affect the system.

What the mind does is only, according to von Neumann, collapse the wave function. It does not make a measurement, as the mind does not have a preferred pointer basis, which is itself provided by the theory of decoherence through super selection rules [33]. In Stapp's model, the mind affects matter by measuring first, say, $|\alpha + \Delta\rangle$, and then $|\alpha + 2\Delta\rangle$, and so on. But the choice of measuring $|\alpha + 2\Delta\rangle$ instead of $|\alpha + \Delta\rangle$ involves the presence of a physical apparatus that measures $|\alpha + 2\Delta\rangle$, physically different from the apparatus that measures $|\alpha + \Delta\rangle$. Such mind may be "observing" this apparatus and making it collapse into one of its pointer basis, with values "yes" or "no," but it cannot make the measurement itself. Thus, we reach the conclusion that, for the measurement to be performed, there must be a way for the mind to affect matter by selecting a specific apparatus and its corresponding pointer basis (instead of another), and we get into a circular argument: to solve the problem of how the mind affects matter, we need to postulate that the mind affects matter. I see this as a serious, perhaps insurmountable, problem for the current approach.

4 Possible Alternatives

I end this paper with possible alternatives on how to approach the issue of consciousness (the easy problem, of course). As Stapp mentioned in a private conversation, one of his motivations for the quantum theory of mind was that it provides an alternative to human decision making being either random (say, as modeled by the classical SR behaviorist theory) or deterministic. He claimed that quantum mechanics, through von Neumann's interpretation, provided an third way: how to think about the decision maker as a free agent (thus nondeterministic) while at the same time not simply coming to decisions by a random process.

Though either alternative seems almost like a (free?) personal choice on how one wants to think about the relationship between consciousness and decision making, I want to provide a fourth way. In between deterministic and stochastic processes, may be stochastically incomplete processes. We know that decision making is highly contextual, in the sense that the probabilistic processes involved in many human decisions cannot be appropriately modeled by classical probability theory [13]. In fact, this stochastic incompleteness is closely related to the quantum mechanical one, in the sense that certain decision-making processes can be better modeled by an algebraic structure inspired by quantum mechanics (see [24,14,10,15,12,3] and references therein). The fact that the mathematical formalism of quantum mechanics leads to better descriptions of social phenomena led to the term *quantum-like* and to new areas of research, such as quantum cognition and quantum finances.

However, as pointed out by many authors [24,14,10,15,12,3], this quantum-like behavior does not mean that such systems are quantum mechanical. What is meant here is simply that the underlying dynamics can be described as a classical one, and yet result in quantum-like effects. In fact, quantum like effects in the brain can be obtained by simple contextual interference [6,7], based on models of neural oscillators that reproduces standard SR theory in certain cases [31,25,9]. This should not be surprising, as realistic models can reproduce the same outcomes of quantum mechanics, as long as no spacelike events are involved or if detectors are not 100% efficient (see [23,26,27,28,29] for one such model).

What is at the core of such quantum-like effects is contextuality. One should not expect social systems to be non-local, as the EPR example, but one should expect them to be contextual. However, if contextuality is the name of the game, then perhaps the quantum mechanical apparatus carries too much baggage with it. For example, one can have stochastically incomplete systems and yet have no quantum description for them [5]. Furthermore, such systems can be modeled by neural oscillators, thus leaving open the idea that using a quantum mechanical description is too constraining. But, more importantly, with alternative descriptions, it is possible to show that further principles can be added, providing a possible constraint in decision making that is neither quantum nor random [8]. This opens up the exciting perspective of having an approach to decision making that satisfies Stapp's criteria, in a certain sense, and is at the same time testable at the behavioral level.

Let me end with a final general comment about using classical approaches to understand something that is, in essence, a quantum phenomena, as quantum mechanics is the ultimate theory of Nature. Physics is not only about constructing theories, but also models. For example, though classical mechanics is essentially wrong, one would not use QM in a model for spaceship trajectories. A quantum model would not only be impractical, but would also not add anything to the "story," to our understanding of the issue. In fact, even the simplest attempts to prove the stability of matter from QM have failed miserably at macroscopic levels (see reference [16] for a review). Models tell us a story of causal (including probabilistically causal) connections. This is what we call understanding in physics (e.g., we understand planetary trajectories because we can tell them from Newton's gravity). More importantly, classical mechanics is a good approximation for QM for most macroscopic objects (including not so macroscopic ones such as neurons). In fact, in many simulations with molecules, Newtonian mechanics works pretty good. What is fundamental then? QFT? Should we try to abandon even the view that there are particles in the brain, and try to model it with fields? Strings? From a pragmatic point of view, we don't use theories because they are more fundamental. We use approximations that allow us to say something about the system in a coherent way, and try to justify such approximations based on reasonable assumptions and empirical data. I hope the theory put forth in the section, though classical, may be of help to elucidate certain aspects of decision making that have eluded explanation.

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