

Quantum mechanics, interference, and the brain

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Abstract

In this paper we discuss the use of quantum mechanics to model psychological experiments, starting by sharply contrasting the need of these models to use quantum mechanical nonlocality instead of contextuality. We argue that contextuality, in the form of quantum interference, is the only relevant quantum feature used. Nonlocality does not play a role in those models. Since contextuality is also present in classical models, we propose that classical systems be used to reproduce the quantum models used. We also discuss how classical interference in the brain may lead to contextual processes, and what neural mechanisms may account for it.

Key words: Quantum mechanics; contextuality; Oscillators; Classical fields; Interference

1. Introduction

The human brain is arguably the most powerful computational device known. The underlying mechanisms behind it are not yet revealed, though progress has been made in recent years toward their understanding. One of the mysteries is how fast the brain processes information, given that neurons are relatively slow. In recent years, there has been an increasing number of researchers speculating that the high processing speed of the brain and the emergence of consciousness are due to quantum processes, perhaps even quantum computations (Ricciardi and Umezawa, 1967; Eccles, 1986; Jahn and Dunne, 1986; Eccles, 1990; Beck and Eccles, 1992; Vitiello, 1995; Hameroff, 1998; Thaheld, 2003; Kurita, 2005; Schwartz et al., 2005; Khrennikov, 2006; Freeman and Vitiello, 2006).

Richard Feynman was one of the first persons to discuss quantum computers. In Feynman (1996), he asked whether there were any advantages if the bits of

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information were treated as quantum mechanical superpositions. Some years after Feynman's remarks, Shor (1999) found a quantum algorithm that could factor a prime number in polynomial time. Since no known classical algorithms factors prime numbers that quickly, Shor's work brought quantum computers to the fore.

Quantum algorithms differ from classical ones because a single quantum system can be represented by the linear superposition of possible orthogonal states. If a particular state is not realized by the system, i.e., the system does not collapse onto it, this state can interfere with other possible states. This interference allows for multiple computations via different paths without the system actually collapsing through those paths. In other words, a quantum computer can work on potential realizations of computations. This should be contrasted with a classical computer, which needs to step through each individual computational path. The consequence is that quantum computers can perform massively parallel "virtual" computations (Steane, 1998).

If the brain uses quantum computations, this could explain why it is so fast. However, quantum computation has a difficulty. Despite a strong push to build complex quantum computers, up to now none have been built. This is due to a phenomenon called environmental decoherence (Omnes, 1994). When a quantum system interacts with the environment, the phase of the state vector changes stochastically. Since interference between different states depends on phase relations, the more environmentally induced phase changes, the less visible interference will be. At some point, if decoherence is too strong, interference disappears (Omnes, 1994). For quantum computers, the loss of coherence increases exponentially with the number of digits. Consequently, experimental realizations of quantum computers have, thus far, involved only a very small number of bits.

Notwithstanding, the perspective of quantum computation in the brain, a device that operates at relatively high temperatures, is tantalizing. For instance, Penrose and Hameroff (Hameroff, 1998; Penrose, 1994, 1989) proposed that quantum computations might be feasible in protected environments of microtubules in the neurons. In a detailed analysis of different environmental sources of decoherence in the brain, Tegmark (2000) pointed out that the time scale for decoherence is orders of magnitude faster than those calculated by Penrose and Hameroff. Hagan et al. (2002) claimed that Tegmark's work did not address correctly the model proposed by Penrose and Hameroff, and if you took into account the correct dimensions at play, the decoherence time computed by Tegmark could be order of magnitude bigger. However, Rosa and Faber (2004) showed that Hagan et al. (2002) did not use Tegmark's equations under the correct assumptions, and thus the decoherence time would indeed be smaller than estimated by Hameroff (1998). In any case, as Davies (2004) points out, there seems to be lots of wishful thinking on both sides of this discussion, and quantum processing in the brain won't be widely accepted until quantum superpositions are shown to exist in some special cases in the brain. As it stands, it seems that even for microtubules, environmental decoherence would happen so quickly as to render it improbable, though not impossible, that the brain

uses any quantum computations. It is hard to imagine any protected region of the brain where quantum interference could occur without fast decoherence. Despite this, there is a large volume of research on quantum aspects of the brain.

In this paper we show that quantum-like effects can be present in the brain without an underlying quantum process. Our argument will be presented in the following way. In Section 2 we briefly describe the characteristics of quantum mechanics that are considered non-classical. In Section 2.4, we discuss the main empirical arguments in favor of quantum effects in the brain, and we stress that their main characteristic is the contextuality of observables. In Section 3, we argue that the contextual outcomes of experiments of Section 2.4 can be modeled by classical interference. We end with some remarks on what might be the origin of interference in the brain.

2. Quantum mechanics and the nature of reality

Quantum mechanics is extremely successful in describing nature. But, more than a century after its initial formulation, the meaning of this description is still a matter of intense debate. The main points of discussion are the following. (i) Nondeterminism; (ii) contextuality; (iii) Nonlocality. In this section we will analyze each point, and bring out the features that we deem relevant to quantum-mechanical models of the brain.

2.1. Nondeterminism

The nondeterministic nature of the atomic world was first pointed out by Rutherford (Pais, 1986). Because the radioactive decay is exponential, Rutherford recognized that it followed from a *memoryless* process, where the instantaneous rate of decay is proportional to the number of radioactive atoms left at the time. Since a decay does not depend on the state of the system on earlier times, the underlying theory of it should be nondeterministic. Bohr used a similar argument to explain the spectral lines of the hydrogen atom, and when Schrödinger used his wave equation to explain those very same spectral lines, Born interpreted the complex quantum wave as a probability density when squared.

When the founders of QM were trying to make sense of it, they were unaware of the subtle but profound differences between determinism and predictability. The experimental data in QM shows that it is not possible to use the current state of the system to predict the outcomes of future experiments. However, the data do not tell us that the underlying dynamics generating the outcomes of the experiments is not deterministic. To make this point clearer, we examine an example (Sinai, 1959, 1970). Let us say we have a billiard table, and on this table a single billiard ball, modeled by a point particle. Let us also assume that the dynamics underlying the movement of the ball is completely deterministic, and given by the Newtonian law that the angle of reflection is equal to the angle of incidence. Sinai proved that if we insert a convex object in the middle of the billiard table, the movement of the ball changes from non-ergodic to ergodic

(Sinai, 1970). Later on, Ornstein (Ornstein and Weiss, 1991) showed that if we allowed for a small error α in the measurement of the phase space, a Sinai billiard with deterministic dynamics would be indistinguishable from one with nondeterministic dynamics. In other words, Sinai's billiard is so unpredictable and chaotic that we cannot tell whether the underlying dynamics generating the trajectories is deterministic or not ¹.

Thus, what the founders of quantum mechanics thought to be a major difference between quantum and classical physics is not necessarily so. The observed unpredictable behavior of quantum phenomena is present in some classical systems. In some sense, the claim that quantum physics was different from classical physics might have originated from confusion between determinism and predictability (Suppes, 1984, 2002). It is possible that the underlying dynamics of quantum phenomena is deterministic, and that it is impossible to distinguish it from a nondeterministic model. In fact, a deterministic model that yields all the same observable predictions of non-relativistic quantum mechanics was proposed by Bohm (1952a,b).

2.2. Contextuality

Early in the 20th Century, it became clear that microscopic particles could sometimes behave like a particle and sometimes like a wave. To describe the dual dynamics of a particle, a wave propagating in space and time was used to represent the wave-like characteristics of the particle. This wave was represented by a complex-valued function $\psi(\mathbf{r}, t)$, and experimental data indicated a wavelength proportional to the particle's momentum. The meaning of the ψ -function was unclear until Max Born proposed that its absolute value, $|\psi(\mathbf{r}, \mathbf{t})|^2$, was the probability density of finding the particle at position \mathbf{r} and time t *if a measurement was made*. Because momentum is associated with wavelength and position with $|\psi|^2$, it follows that the values of momentum and position for a particle are not simultaneously well defined (von Neumann, 1932/1996). This is reflected in the canonical quantization rule that replaces the classical dynamics generated by the Poisson brackets with the commutator of the corresponding operators (Dirac, 1982, 2001). Let us consider a classical system satisfying the equations with Hamiltonian H :

$$\frac{dp}{dt} = \{p, H\}, \quad (1)$$

$$\frac{dq}{dt} = \{q, H\}, \quad (2)$$

where $H = H(q, p)$ is the Hamiltonian, q the generalized coordinate, and p the canonically conjugated momentum to q , i.e. $\{q, p\} = 1$. To find the dynamics of the corresponding quantum system, we apply the canonical quantization rule.

¹Sinai's billiard can be used to show a stochastic behavior of photon trajectories in a realistic model of quantum mechanics. See, for example, Suppes and de Barros (1994a,b, 1996).

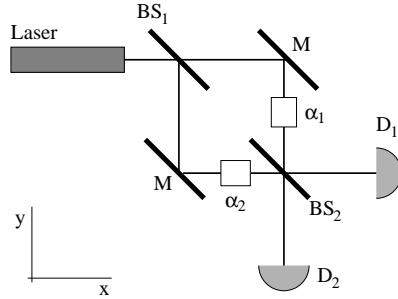


Figure 1: Context-dependent light detection.

First, we substitute for each dynamical variable a quantum observable corresponding to it, i.e., $q \rightarrow \hat{Q}$, $p \rightarrow \hat{P}$, and $H \rightarrow H(\hat{Q}, \hat{P})$. Then we replace the Poisson brackets in (1) and (2) for the commutator times $i\hbar$. Equations (1) and (2) become

$$\begin{aligned} \frac{d\hat{P}}{dt} &= i\hbar [\hat{P}, \hat{H}], \\ \frac{d\hat{Q}}{dt} &= i\hbar [\hat{Q}, \hat{H}]. \end{aligned}$$

Finally, to preserve the symplectic algebra of the Poisson brackets, we also need to impose that $[\hat{Q}, \hat{P}] = i\hbar$. Since in an experiment we can only obtain eigenvalues of the observable, when a measurement is performed the wave vector collapses to one of its eigenvectors (von Neumann, 1932/1996). It follows from the noncommutativity of \hat{Q} and \hat{P} that there are eigenvectors of \hat{Q} that cannot be eigenvectors of \hat{P} . Thus, if we measure \hat{Q} , and then \hat{P} , and then \hat{Q} again, we may obtain a different result for the second measurement of \hat{Q} . The act of measuring \hat{P} “destroys” the properties associated with \hat{Q} .

Though the noncommutativity of observables is a surprising characteristic of quantum mechanics, we want to make a point that there is nothing “spooky” about it. In fact, there are many cases of observables that change their values depending on the order in which we measure them, both inside and outside of physics. For instance, asking the same questions in different orders sometimes yield different response distributions (Weinberger et al., 2006). But, more importantly, the contextuality of measurements exist in classical physics. To see this, let us spell out an example using classical coherent light. The apparatus we will be using is sketched in Figure 1 A monochromatic electromagnetic field generated by a laser can be thought of as a classical field. If we disregard polarization, we can represent this field propagating in the vacuum and in the $\hat{\mathbf{x}}$ direction as

$$E(\mathbf{r}, t) = A \sin(\omega t + \mathbf{k} \cdot \mathbf{r}). \quad (3)$$

In (3), $\omega/2\pi$ is the frequency, and $\mathbf{k} = k\hat{\mathbf{x}}$ is the wave vector, with $k = \omega/c$. From (3), we see that the electric field arriving from the laser at the beam

splitter BS_1 is $E(\mathbf{r}_1, t) = A \sin(\omega t + \theta)$, where $\theta = \mathbf{k} \cdot \mathbf{r}_1$ is a fixed phase. The 50–50 beam splitter BS_1 then splits the light into two beams of equal intensity. When reflected, light acquires an additional phase of $\pi/2$, and after BS_1 we have

$$E(\mathbf{r}, t) = \frac{A}{2} \sin(\omega t + \mathbf{k} \cdot \mathbf{r} + \theta) + \frac{A}{2} \sin(\omega t - \mathbf{k}' \cdot \mathbf{r} + \theta + \frac{\pi}{2}),$$

where $\mathbf{k}' = k\hat{\mathbf{y}}$. Upon reflection on the mirrors M , and passing through the phase shifters α_1 and α_2 and thereafter the second beam splitter BS_2 , the fields at D_1 and D_2 become

$$E(\mathbf{r}_{D_1}, t) = \frac{A}{4} \sin(\omega t + \theta' + \alpha_1 + \pi) + \frac{A}{4} \sin(\omega t + \theta' + \alpha_2 + \pi), \quad (4)$$

and

$$E(\mathbf{r}_{D_2}, t) = \frac{A}{4} \sin(\omega t + \theta' + \alpha_1 + \frac{\pi}{2}) + \frac{A}{4} \sin(\omega t + \theta' + \alpha_2 + \frac{3\pi}{2}). \quad (5)$$

In the above equations, we assumed that the interferometer in Figure 1 has arms of equal length, and therefore the distance dependent phase factor coming from the term $\mathbf{k} \cdot \mathbf{r}$ is the same for all beams reaching D_1 and D_2 . To compute the light intensity reaching D_1 and D_2 , it is easier if we rewrite (4) and (5) in complex notation, and they become

$$E(\mathbf{r}_{D_1}, t) = \frac{A}{4} \left(e^{i(\omega t + \alpha_1 + \pi)} + e^{i(\omega t + \alpha_2 + \pi)} \right),$$

and

$$E(\mathbf{r}_{D_2}, t) = \frac{A}{4} \left(e^{i(\omega t + \alpha_1 + \pi/2)} + e^{i(\omega t + \alpha_2 + 3\pi/2)} \right).$$

If we now compute the intensities of the fields I_1 and I_2 at D_1 and D_2 , we obtain at once

$$\begin{aligned} I_1 &= |E(\mathbf{r}_{D_1}, t)|^2 \\ &= \frac{A^2}{8} \left(2 + e^{i(\alpha_1 - \alpha_2)} + e^{-i(\alpha_1 - \alpha_2)} \right) \\ &= \frac{A^2}{4} (1 + \cos(\alpha_1 - \alpha_2)), \end{aligned}$$

and

$$I_2 = \frac{A^2}{4} (1 - \cos(\alpha_1 - \alpha_2)).$$

That this result shows contextuality follows from Bell's inequalities, as it is straightforward to show that the correlations between I_1 and I_2 violate them.

Bell's inequalities were proposed in the context of quantum mechanical non-locality (Einstein et al., 1935; Bell, 1966; Clauser and Shimony, 1978). However, Bell's inequalities have a meaning that is independent of Physical models. To understand its meaning, let us start with three ± 1 -valued random variables, \mathbf{A} ,

Run	A	B	C
1	-1	·	-1
2	1	·	-1
3	·	·	1
4	·	-1	·
5	1	1	·
6	·	·	1
7	⋮	⋮	⋮

Table 1: Hypothetical experiment showing the outcomes of **A**, **B**, and **C**. The outcome of each measurement is either +1 or -1. If a variable is not measured, this is represented by a “·”.

B, and **C**, each with expectations equal to zero (e.g., $E(\mathbf{A}) = 0$). Let us further assume that the experimental setup is such that we cannot measure the three random variables simultaneously, but only in pairs. An example of how data from such variables could look like is shown in Table 1. Clearly, the only directly measurable correlations are $E(\mathbf{AB})$, $E(\mathbf{AC})$, and $E(\mathbf{BC})$. So, it is reasonable to ask, in some situations, whether we can infer the value of the triple moment $E(\mathbf{ABC})$. In other words, can we find values for the missing data on Table 1 that are consistent with the measurable correlations? The answer is yes, if the inequalities

$$-1 \leq E(\mathbf{AB}) + E(\mathbf{AC}) + E(\mathbf{BC}) \leq 1 + 2 \min \{E(\mathbf{AB}), E(\mathbf{AC}), E(\mathbf{BC})\} \quad (6)$$

are satisfied. If, for some reason, the experimental data violate the above inequalities, it is not possible to fill the data table with the missing values in a way consistent with the observed correlations. In other words, the random variables **A**, **B**, and **C** do not have a joint probability distribution (Suppes and Zanotti, 1981).

To clarify the meaning of the nonexistence of a joint probability distribution and its relationship to contextuality, let us look at the Theorem of Common Causes, proved by Suppes and Zanotti (1981). This theorem states that, for a set of two-valued random variables, it is always possible to find a common cause λ that factorizes their probabilities if and only if there exists a joint probability distribution. In other words, we can find a λ such that

$$P(\mathbf{A} = 1, \mathbf{B} = 1, \mathbf{C} = 1) = P(\mathbf{A} = 1|\lambda = \lambda) P(\mathbf{B} = 1|\lambda = \lambda) P(\mathbf{C} = 1|\lambda = \lambda)$$

if **A**, **B**, and **C** have a joint probability distribution. Now, if a joint probability does not exist, a common cause λ that can explain the correlations cannot exist either. However, pairwise joints exist, as we measure the correlations between **A**, **B**, and **C**. Thus, for each pair, we can construct a λ , say $\lambda_{\mathbf{AB}}$, $\lambda_{\mathbf{BC}}$, and $\lambda_{\mathbf{AC}}$. We can think of each of those pairwise common causes as related to the context of the measurement (Svozil, 2005). Thus, the nonexistence of a common cause for all measurements implies the nonexistence of a common context for

all correlations, and we can say that if there is no joint probability distribution we have contextuality.

So, the fact that I_1 and I_2 , as above constructed, violate Bell's inequalities means that they do not allow for the existence of a joint probability distribution. A reader familiar with Bell's inequalities might argue that I_1 and I_2 are continuous variables, and that (6) was derived for ± 1 -valued random variables. This is not in itself a problem, since there can then be no hidden variable that factors out the correlations conditionally because the correlation matrix for I_1 and I_2 is not nonnegative definite (Suppes and Zanotti, 1981; Holland and Rosenbaum, 1986). But if we wish to obtain correlations that are discrete, all we need is to associate to each intensity a random variable corresponding to the detection of a single particle (in this case a photon), and to conditionalize the measurement to two-photon detections (Suppes et al., 1996a). Since a violation of Bell's inequalities imply the nonexistence of a joint probability distribution Suppes and Zanotti (1981), this classical field example shows a contextual measurement. The context in this case comes from choosing two sets of angles, α_1 , α'_1 , α_2 , and α'_2 . Since we can't measure the field with the settings α_1 and α'_1 simultaneously, we need to choose which one to measure, and its value is affected by those of α_2 and α'_2 (the context).

An alert reader may object to the choice of excluding events that produced two particle detections, as this could require superluminal communication for spacelike separated events. We emphasize that given the dimensions of the brain and the characteristic times involved in cognitive computations, we are very skeptical that superluminal correlations of spacelike separated events are relevant in the brain. So, for our purposes no superluminal signaling would be required. However, even without superluminal signaling, a local model of photons that violate Bells inequalities can be constructed using ideas similar to those shown above (see Suppes, de Barros, and Sant'Anna (1996b) for an example).

2.3. Non-locality

The main question regarding the interpretation of quantum mechanics revolves around the meaning of a state vector or wave function. One way to approach this problem is to say that this vector, whose projected absolute value gives the probability of an observable, is a representation of hidden states that, when averaged, form the "stuff" that we measure. A theory that uses underlying physical objects that would yield the outcomes of measurement with the same probabilities predicted by quantum mechanics is known as a *hidden-variable theory*. The contextuality of some quantum mechanical measurements implies, as we discussed above, that there are no hidden variables that can explain the outcomes of those experiments².

²There are many examples of hidden-variable theories that reproduce the outcomes of quantum mechanics. A historically important one, as it was the first hidden-variable theory to predict the same outcomes as quantum mechanics, is Bohm's interpretation of quantum

But to physicists, the main problem with quantum mechanics is that it is not possible to find a set of common hidden variables that factor the outcomes of experiments that are separated by a *spacelike* interval. This nonlocality is unacceptable, since it seems to conflict with special relativity. In fact, nonlocality was the reason behind the rejection by many physicists, including Einstein, of Bohm's theory, and for many years physicists tried to construct a local hidden-variable theory. However, in his seminal paper, John Bell (1966) showed that such a theory was not compatible with the predictions of quantum mechanics. Experiments conducted by Aspect and collaborators (Aspect et al., 1982a,b) showed that quantum mechanics was indeed correct, burying for many the possibility of a local hidden-variable theory³.

2.4. Quantum Mechanics and the Brain

As we mentioned in Section 2, the nondeterministic aspect of quantum mechanics was considered by many physicists to be a major rupture with classical Newtonian physics and a unavoidable characteristic of the microscopic world. Before we talk about the relationship between quantum mechanics and the brain, it is important to discuss how nondeterminism appears in the theory. Quantum mechanical systems are described by a state vector belonging to a Hilbert space. In the simple case of a single particle, this vector is often represented by a complex wave in three-dimensional space. Under "normal" conditions, the evolution of a quantum system is described by Schrödinger's equation

$$i\hbar \frac{d}{dt} |\psi\rangle = \hat{H} |\psi\rangle,$$

where \hat{H} is the Hamiltonian operator. Schrödinger's equation is a first-order differential equation, and the state of the system at time t_0 completely determines the state at time $t > t_0$. So, where is quantum mechanics nondeterministic? The answer to this question, according to Bohr, lies in the measurement process. When we measure an observable \hat{A} , the original state vector collapses to one of the eigenvectors of the observable operator. This collapse happens in a nondeterministic way, with the probabilities for each collapse given by the

mechanics. In it, the quantum behavior of a particle is explained by the effects of a pilot wave (Bohm, 1952a,b). A reader familiar with this theory may be confused by our claim of non-existence of hidden variables. We point out that in the single-particle version of Bohm's theory the hidden variable itself is contextual, as the pilot wave must satisfy the different boundary conditions imposed by different measuring apparatus. Furthermore, for more than one particle, the theory is not only contextual, but highly nonlocal.

³Many other theories or interpretations of quantum mechanics exist, like the many worlds (Everett III, 1957), Nelson's stochastic mechanics (Nelson, 1985), the prism models (Fine, 1982a,b; Szabo and Fine, 2002; Everett III, 1957), or t'Hooft's theory (t'Hooft, 2001). But, eventually, those theories need to address the issue of non-locality. Often they do so by either postulating some unmeasurable observables, like prism models, and thus relying on measurement inefficiency, or simply by having underlying non-local interactions, like Nelson's mechanics.

square of the absolute value of the coefficients of the decomposition of the vector into the eigenstates of \hat{A} (von Neumann, 1932/1996). In other words, the evolution of a quantum system is deterministic most of the time, *except when a measurement is made*.

A measurement, though, is the interaction of a measuring apparatus with the system being measured. This interaction happens in such a way that the final state of the measuring apparatus correlates to the value of the measured property of the system. Of course, the state of the system composed of the measurement apparatus and the measured object should itself be described by a state vector, and its evolution should follow Schrödinger's equation. This presents a problem; nowhere in this description is there room for a probabilistic collapse of the state, since the evolution is dictated by a deterministic equation. To introduce this nondeterminism, we have to introduce another measurement apparatus that measures our original measurement apparatus, and hence we fall into an infinite regression. This need of a measurement of the measurement apparatus is what is historically known as the measurement problem.

To tackle the measurement problem, some physicists started asking when the infinite regression of measurements would end. To some, there was a natural final measuring device: the human mind. This led some prominent physicists to seek a connection between the brain and quantum physics. The early focus for this connection was, and still often continues to be, on the measuring process. Niels Bohr was one of the first to point out the apparently dual nature of a macroscopic observer measuring and thereby disturbing in some sense, by the very physical nature of the measurement process, the quantum system being observed (Pais, 1986). Some physicists, like Erwin Schrödinger (1996) and Bohm (1990), even proposed a dualistic view of mind and brain to address the measurement problem. But one of the most prominent current proponents of brain and quantum mechanical connection, Roger Penrose, is not sympathetic to "dualistic mind" views: "In my own opinion, it is not very helpful, from the scientific point of view, to think of a dualistic 'mind' that is (logically) *external* to the body, somehow influencing the choices that seem to arise in the action of \mathbf{R} [state vector reduction]. If the 'will' could somehow influence Nature's choice of alternative that occurs with \mathbf{R} , then why is an experimenter not able, by the action of 'will power', to influence the result of a quantum experiment? If this were possible, then violations of the quantum probabilities would surely be rife! For myself, I cannot believe that such a picture can be close to the truth. To have an external 'mind-stuff' that is not itself subject to physical laws is taking us outside anything that could be reasonably called a scientific explanation" (Penrose, 1994, p. 350). We agree with Penrose on the issue of dualism.

Though historically the connection between quantum mechanics and the brain started with the measurement problem, nowadays lots of attention has been focused on the brain as a quantum computer. The brain's extraordinary computational power led several scientists, Penrose included, to suggest that it uses quantum computation, as we mentioned in Section 1. We will not examine this topic in detail, but a good informal yet careful review of the reasons for skepticism about this claim has been given recently by Koch and Hepp (2006).

Some detailed negative arguments based on the rapid decoherence process of entangled quantum particles in most environments are to be found in Tegmark (2000). So we shall not explore here the issue of decoherence. Instead, we approach the relation between quantum phenomena and the brain by asking what kinds of quantum computations are often proposed, and to see if those computations can be reproduced by classical processes. This is the main goal of the next section.

3. Fields, oscillators, and the brain

Though quantum mechanics received lots of attention with Penrose's proposal that quantum computation is related to consciousness, other researchers see quantum mechanics as a possible mechanism for other cognitive processes. For example, Khrennikov and Haven (2007) claim that quantum probability interference is present in social phenomena as well as in cognition. In a more detailed and, in our opinion, interesting paper, Busemeyer et al. (2006) analyzed the dynamics of human decision-making, and showed that not only purely Markov models didn't fit the data well, but a better fit could be achieved by using quantum dynamics. Because of its better fit to the data and the straightforward distinction between quantum and classical dynamics made by Busemeyer et al. (2006), we will discuss this work in some detail.

3.1. Interference

Our goal will be to show that indeed interference is an important factor because of the contextuality of the observers. In resonance with our earlier remarks, it is our claim that quantum mechanics is not necessary to reproduce the kind of contextuality needed for brain processes. The reason is the lack of a need to correlate contextual measurements made in spacelike separated regions of spacetime. Such contextuality could then, in principle, be accounted for by classical interference. So, in this section, we will construct a dynamic model using classical interference that reproduces the quantum dynamics used by Busemeyer et al. (2006).

3.2. The Quantum Mechanical model

We start with Busemeyer et al. (2006) analysis of quantum dynamics of a state that undergoes successive transformations. Let us begin with $|\psi\rangle \in \mathcal{H}$, where \mathcal{H} is the Hilbert space representing the system (von Neumann, 1932/1996; Dirac, 1982; Cohen-Tannoudji et al., 1977). The process is represented by Figure 2. Since D_1 and D_2 are detectors that measure the system, we would like to know how to determine the probabilities $P(D_1)$ and $P(D_2)$ of observing the system at D_1 and D_2 .

We note that a similar concept to this concept of interference in classical optics has been used in psychological phenomena of learning since early in the 20th century, if not earlier. The basic idea is that past learning can interfere with learning a new related concept or behavior that has serious overlap with

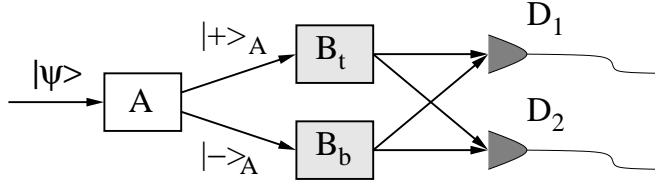


Figure 2: Successive measurements of a state $|\psi\rangle$. The state $|\psi\rangle$ goes through a device A that split it into two new orthogonal states, denoted $|+\rangle_A$ and $|-\rangle_A$. The two states are then fed into two identical devices, B_t and B_b , and each device split the beams into two new orthogonal states, $|+\rangle_B$ and $|-\rangle_B$, such that they are recombined, as shown, at detectors D_1 and D_2 . We assume that the observables associated to A do not commute with B .

the old. As an example studied experimentally in Suppes (1965), children at about the age of five years old can learn rather easily when two finite sets are identical. But this learning interferes with learning the concept of two sets being equivalent, i.e., having the same number of members. A neural network that models this result is given in Suppes and Liang (1998). In the psychological literature, interference is often labeled negative transfer, with a corresponding meaning attached to positive transfer, as in positive interference familiar in physical optics.

Probabilities with interference. The vector state $|\psi\rangle$ that enters A comes out of it as a superposition of the two states

$$e^{i\theta_t} \sqrt{a_+} |+\rangle_A + e^{i\theta_b} \sqrt{1-a_+} |-\rangle_A.$$

Since the state vector $|+\rangle_A$ ($|-\rangle_A$) goes into B_t (B_b), it also splits into the final states going to the detectors, and we have

$$\begin{aligned} e^{i\theta_t} \sqrt{a_+} |+\rangle_A + e^{i\theta_b} \sqrt{1-a_+} |-\rangle_A &\rightarrow a_+ e^{i\theta_t} \left(\frac{\sqrt{1-a_+}}{\sqrt{a_+}} |D_1\rangle + |D_2\rangle \right) + \\ &(1-a_+) e^{i\theta_b} \left(\frac{\sqrt{a_+}}{\sqrt{1-a_+}} |D_1\rangle + |D_2\rangle \right). \end{aligned}$$

Rewriting the right hand term, we get

$$(e^{i\theta_t} + e^{i\theta_b}) \sqrt{a_+} \sqrt{1-a_+} |D_1\rangle + (e^{i\theta_t} a_+ + e^{i\theta_b} (1-a_+)) |D_2\rangle.$$

The probability $P(D_1)$ is then given by

$$\begin{aligned} P(D_1) &= |(e^{i\theta_t} + e^{i\theta_b}) \sqrt{a_+} \sqrt{1-a_+}|^2 \\ &= 2a_+ (1-a_+) [1 + \cos(\theta_t - \theta_b)]. \end{aligned}$$

This expression clearly shows an interference term reflecting the phase differences in the superposition. In fact, without changing the probabilities of observations of B_b and B_t , we could change the final probabilities of D_1 and D_2 by introducing phase parameters θ_i that would change the interference patterns

at the detectors. These interference effects are not present in the Markovian models, but form the core of the quantum mechanical models discussed by Busemeyer et al. (2006).

Probabilities with environmental decoherence. No system is truly isolated, and the interaction of a quantum system with the surrounding environment leads to a process called environmental decoherence (Omnes, 1994). The consequence of decoherence for our quantum model described above is that the phase relations quickly disappear. So, as before, immediately after A , the state becomes

$$e^{i\theta_t} \sqrt{a_+} |+\rangle_A + e^{i\theta_b} \sqrt{1-a_+} |-\rangle_A,$$

where θ_t and θ_b are phases that may be introduced by some physical process. However, as the system interacts with the noisy environment, the above state evolves into a proper mixture of $|+\rangle_A$ and $|-\rangle_A$. The probabilities are given by the absolute value of the coefficients, i.e. $P(|+\rangle_A|\psi\rangle) = |e^{i\theta_t} \sqrt{a_+}|^2 = a_+$ and $P(|-\rangle_A|\psi\rangle) = |e^{i\theta_b} \sqrt{1-a_+}|^2 = 1-a_+$. As we can see from the conditional probabilities just computed, there is no dependence on the phases θ_b and θ_t . Once the system reaches B_t or B_b , we have a new evolution of the states, until they reach D_1 and D_2 , and we also have those probabilities not depending on phase due to decoherence. If we set $P(D_1|+\rangle_A) = 1-a_+$, $P(D_2|+\rangle_A) = a_+$, $P(D_1|-\rangle_A) = a_+$, and $P(D_2|-\rangle_A) = 1-a_+$, the probability $P(D_1)$ follows from a direct computation as

$$P(D_1) = 2a_+(1-a_+). \quad (7)$$

Similar computations can be carried out for D_2 .

It is interesting to note that the probability in (7) is similar to the Markovian model presented by Busemeyer et al. (2006), as well as their quantum model with measurement. This should not come as a surprise, since decoherence is responsible for the classical behavior of quantum systems.

3.3. Classical Oscillator Model

Classical fields yield results that are similar to the quantum mechanical ones when we have no decoherence. Since we are not interested in the field propagation dynamics, we will focus only on its behavior in time at a fixed point. Thus, in this model we replace the state $|\psi\rangle$ with a field oscillating in time, represented by the oscillator $\psi(t) = e^{i\omega t}$. The intensity of oscillation is normalized, i.e. $I = \psi^*(t) \cdot \psi(t) = 1$. We can model each measurement as a beam splitter, such that the beam going to B_t is represented by the oscillators $\psi_+(t) = \sqrt{a_+} e^{i(\omega t + \theta_t)}$ and to B_b is $\psi_-(t) = \sqrt{1-a_+} e^{i(\omega t + \theta_b)}$, where θ_t and θ_b are adjustable phases added to the field. If we measure the field at each branch, B_t and B_b , we obtain a_+ and $1-a_+$. We can, if we wish, define ± 1 -valued random variables representing a detection at B_t and B_b , and have them proportional to a_+ and $1-a_+$, thus reproducing the same behavior as the Markov process. Finally, at each node B_t and B_b , the field splits again, and new phases are added. If we look at D_1 , we would observe the following.

$$F_{D_1} = \sqrt{a_+} \sqrt{1-a_+} e^{i(\omega t + \theta_t)} + \sqrt{1-a_+} \sqrt{a_+} e^{i(\omega t + \theta_b)}.$$

The intensity is

$$\begin{aligned} I_{D_1} &= |F_{D_1}| \\ &= 2a_+(1 - a_+) [1 + \cos(\theta_t - \theta_b)]. \end{aligned} \quad (8)$$

The $\cos(\theta_t - \theta_b)$ is the typical interference term, similar to the quantum mechanical one. Once again, we could associate to detectors D_1 and D_2 ± 1 -valued random variables and reproduce the quantum-mechanical measurements.

To model with oscillators a Markovian process, we would need to remove all interference effects. With oscillators, this can be done by adding to each step that corresponds to a quantum measurement, say B_t , a stochastic phase represented by a random variable θ uniformly distributed on the interval $[0, 2\pi]$. Because at each run the phase relations change, the mean of the cos term in Equation 8 is zero, and the probability $P(D_1) \propto 2a_+(1 - a_+)$ would reduce to the Markovian one, with no interference term.

Other purely probabilistic models using oscillators can be constructed. For example, in Suppes and de Barros (1994b,a), a random-walk model was used to produce interference patterns with particles with well-defined trajectories. Even though the particles used in the above articles are far from what one would consider classical, they only interacted locally and probabilistically. Another possible approach could be that of Khrennikov (2005); Conte et al. (2007), where each context would require the definition of a set of classical-like variables. Nevertheless, these approaches are meant to reproduce quantum mechanical results, and are significantly richer than the simple model presented above. In fact, we should point out that, contrary to Suppes and de Barros (1994b,a); Khrennikov (2005); Conte et al. (2007), the oscillator approach shown above is purely classical. A classical approach is possible because we can choose frequencies ω that are very small compared to frequencies that would be associated to superluminal (and therefore nonlocal) processes. In other words, to have non-locality, the brain would have to operate with frequencies of the order of GHz, which we believe is an unreasonable assumption.

4. Final Remarks

In a previous paper (Suppes and de Barros, 2007), we analyzed quantum effects in the brain from a different perspective, as a consequence of an eye photodetector being able to measure single photon states. In this paper we continued the analysis by making a distinction between nonlocality and contextuality, and asking if quantum effects are really necessary beyond the ones presented in (Suppes and de Barros, 2007). In some sense, nonlocality and contextuality are intimately related, since the former requires the latter to happen with contexts that are changed in a spacelike way. However, when taken by itself, contextuality is not a surprising phenomena, but a rather common one, whereas nonlocality is still a disturbing characteristic of quantum mechanics.

In our opinion, what is interesting about works like Busemeyer et al. (2006) and Khrennikov and Haven (2007) is not their use of quantum dynamics, but the

implied contextuality of observables that is a consequence of quantum interference. Our claim is that they seem to show compelling evidence that interference may play an important role in cognitive processes, but that this interference need not be of quantum origin.

If we have interference, then what is its origin? We believe there is a natural mechanics in the brain for this type of interference: neural oscillators. There is evidence that neuron synchronization plays an important role in higher cognitive processes. Neural oscillators, made up of collections of synchronized neurons are apparently ubiquitous in the brain, and their oscillations are macroscopically observable in electroencephalograms (Gerstner and Kistler, 2002; Freeman, 1979; Wright and Liley, 1995). These neural synchronizations happen because of oscillations around equilibrium points in the phase space of neural networks (Izhikevich, 1999, 2007). Detailed theoretical analysis of weakly interacting neurons close to a bifurcation show oscillations (Gerstner and Kistler, 2002; Hoppensteadt and Izhikevich, 1996a,b; Izhikevich, 2007). Cortical oscillations may propagate in the cortex as if they were waves (Nunez and Srinivasan, 2006). Furthermore, synchronized cortical oscillations in different regions of the cortex are related to some cognitive processes and perhaps even consciousness (Bush and Sejnowski, 1996; Massimini et al., 2005; Ritz and Sejnowski, 1997; Izhikevich, 1999; Kazanovich and Borisyuk, 2002; Srinivasan et al., 1999; Tallon-Baudry et al., 2001; Tonnelier et al., 1999; Tononi and Edelman, 1998; Ward, 2003). Many experiments not only show the presence of oscillators in the brain (Eckhorn et al., 1988; Friedrich et al., 2004; Kazantsev et al., 2004; Lutz et al., 2002; Murthy and Fetz, 1992; Rees et al., 2002; Rodriguez et al., 1999; Sompolinsky et al., 1990; Tallon-Baudry et al., 2001; Steinmetz et al., 2000; Wang, 1995), but also that their synchronization is related to perceptual processing (Friedrich et al., 2004; Kazantsev et al., 2004; Leznik et al., 2002; Murthy and Fetz, 1992; Sompolinsky et al., 1990) and may play a role in solving the binding problem (Eckhorn et al., 1988). Neural oscillators have already been used to model a wide range of brain functions, such as pyramidal cells (Lytton and Sejnowski, 1991), effects of electric fields in epilepsy (Park et al., 2003), activities in the cat visual cortex (Sompolinsky et al., 1990), learning of songs by birds (Trevisan et al., 2005), and coordinated finger tapping (Yamanishi et al., 1980). Suppes and Han (2000) showed that a small number of frequencies can be used to recognize a verbal stimulus from EEG data, consistent with the brain representation of language being neural oscillators. Our claim is that the neural oscillations may play the role of a wave source, with a wave propagating to another cortical region and interfering with other sources. This type of interference would be contextual, in the sense described above, and would be an important process in cognitive computations by the brain.

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